

# Mathematical Modelling for Improved Blood Flow in a Sick Cell Anaemia Patient with Morphological Effect

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**Abstract:** Sick cell is a disease that affects the growth and life expectancy of a given population infected with this disease. Hence, we carried out a theoretical study on the improvement of blood flow and the morphology effect on red blood cells in sickle cell patient using a mathematical model. This morphological effect on the red blood cell comes as a result of the effect of treatment parameter embedded in the governing equation. The governing dimensional second order partial differential equations was transformed to non-dimensional form and solved analytically using the Frobenius method and solutions was gotten for both the blood momentum, energy and diffusion. The solutions for the flow of the red blood cell and wall shear stress was obtained with the result showing that heat source increase causes an increase in the flow of blood, reducing the shear stress at the wall and increasing the volumetric flow rate. This effect caused an improvement in the sickle shape of the deformed RBC and an improved flow which will reduce the crises experienced in patients with SCD. Finally, the increase in chemical reaction caused an increase in the pulsatile pressure of the sickled blood cell which results to an increase in the blood flow.

**Keywords:** Hemoglobin, Pulsatile Pressure, Heat Source, Chemical Reaction, Blood Flow, Wall Shear Stress

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## 1. Introduction

The erythrocytes or matured red blood cells (RBC), are flexible solution that are saturated whose weight is approximated to be 32% of hemoglobin in 65% water together with elements that are inorganic (K, Na, Mg and Ca). The protein found in RBC is referred as the hemoglobin which gives a reddish colour to the blood with the involvement of transportation of oxygen and carbon-dioxide between the body lungs and body tissues. A blood disorder that is hereditary is called a sickle cell disease (SCD) [1] caused by a point mutation occurring at the gene of b-globin, resulting to a substitution of Glu  $\rightarrow$  Val in the protein, producing a b-globin (bS) that is sickles in hypoxic situation.

Mutation may not have an initial effect on the fetus in humans, since there is an expression of the c-globin in the RBC which dominates the fetal blood, which causes a predominance of the fetal hemoglobin (HbF) [2]. Certain forces could actually help to restore the deformed red blood cell which becomes sickle cell taking a crescent moon like shape that is sticky and rigid which reduces or hinders blood flow. These forces are the electromagnetic forces, buoyance force, slip and pressure force, with the inclusion of both chemical reaction and energy source. Hence we shall be looking at how these forces applied on the boundary of the RBC will help in returning the shape memory to its normal biconcave shape.



Figure 1. Diagram showing normal and sickled red blood cells.



Figure 2. Diagram showing the theoretical shape of the normal red cell [3].

Sickle-cell disease is well documented as one of the most common severe monogenic disorders in the world. The main pathophysiology of this disease is vaso-occlusion and polymerisation of hemoglobin leading to erythrocyte rigidity, David Rees *et al* [4]. Sick cell anaemia is a genetic defect that causes the red blood cells to become sickle in shape Weatherall and Clegg [5]. Due to inadequate amount of oxygen transported to peripheral tissue by these sickle cells, many organs are affected due to reoccurring inflammation and vaso-occlusion which causes an increase in brain damage, cardiovascular system, lungs, kidneys and bones, which is more prevalent as the age increases resulting to the body suffering the effects. These inadequacies which bring about what is called sickle cell disease (SCD). A major complication from sickle cell disease is Cerebrovascular accident (CVA) which results to hemorrhagic stroke, Kwaku *et al* [6]. Literature holds many contributions from researchers all over the world for the management and treatment of this disease. The Management of the disease is aimed at preventing complications and the frequent pain crises. Treatments include medications, blood transfusions and stem cell transplant for teenagers. Amongst many researchers, Athiwat Hutchaleelaha *et al* [7] in a work titled 'pharmacodynamics and pharmacokinetics of voxelotor (GBT440) in adults that are healthy and patients who have a sickle cell disease' showed the tolerance and efficacy of voxelotor in the treatment of SCD. Biomechanical forces are generated in blood circulation, the obviously known forces are shear, stress, circumferential stretch and area in focus, and these directly act on the endothelium. The spherical nature of normal blood cells helps make this process smooth. During the development of the embryo, endothelial cells (ECs) maintain a tight barrier function and adapts to the growth of the vascular tree that is created and remodelled, Pedro Campinho *et al* [8]. The understanding of these biomechanical forces helps in implementing Shear-mediated platelet activation (SMPA), at the central in thrombosis of cardiovascular therapeutic devices that is implantable. Marvin Slepian *et al* [9]. The normal stress is the ones with diagonal component while shear stress is the one with horizontal component.

Eldesoky [10] proposed an ordinary differential equations mathematical model of blood flow that is parallel with applied magnetic field in the transverse direction with effect of heat source. Sen and Chakravarty [11] did a study on the constricted flow phenomena in arteries using mathematical

model appropriately. Shit and Roy [12] did a theoretical study on the flow of blood that is pulsatile through a channel that is porous and constricted with the effect of a magnetic field with the blood considered to be an incompressible Newtonian fluid. Mukesh *et al.* [13] did a study on the influence of transvers magnetic field on a blood flow that is unsteady and pulsatile flowing through an artery with stenosis using a mathematical model. Kumar *et al.* [14] did a study using a mathematical model to show magneto-hydrodynamic effects on the flow of blood through a porous inclined tapered artery. Sinha *et al.* [15] did a study on the effect of the transfer of heat on permeable vessel of unsteady Magneto-hydrodynamic blood flow with non-uniform heat source present. Vincent *et al.* [16] did a study on the flow of blood fluid that is viscous, incompressible and conducting electricity in a circular tube that is rigid and inclined with the effect of magnetic field. Sharma *et al.* [17] studied the effect of variable viscosity and slip parameter on magneto-hydrodynamic flow of blood and transfer of heat through a medium that is porous. The magnetic field applied to the porous medium is placed perpendicularly to the surface. Karthikeyan and Jeevitha [18] did a study analysis the effect of heat and mass transfer on a two phase model of an unsteady blood flow that is pulsatile past an artery with stenosis having a wall that is permeable with radiation and chemical reaction effects. Chinedu and Amadi [19] used the Frobenius method to obtain analytical solution of a Non-isothermal flow in a cylindrical geometry. No slip was considered and an analytical solution in closed form was gotten for the flow variables.

Red blood cells could take sickled form as a result of morphological changes that may have occurred. Certain boundary conditions could affect the shape of the RBC either deforming it or reforming it. Hasitha *et al.* [20], studied red blood cell deformation using numerical method. The RBC membrane was represented using two dimensional spring network such that the compressed energy, elastic stretch and the bending energy was considered with the surface area having constant constraints. The red blood cells had forces acting on it gotten from virtual work principles with the discretization of the finite number of fluid particles using smoothed particles.

Timothy and Philip [21] described the Red blood cells where morphological changes was gotten naturally under patho-physiological situations such as echinocytes and stomatocytes using two mathematical models. The models represented the changes in the shape of the RBC during experimental observation with magnetic resonance micro imaging used to detect the changes that occurred. Paul *et al.* [22] computed morphological deformation of the red blood cell from photonic stress distribution past a membrane that is cellular using numerical based method on a theory of an elastic membrane. Holes law was applied with the energy of the cell membrane thickness obtained applied during experimental conditions. Yixiang *et al.*, [23] did a computational and experimental study quantifying mature sickled erythrocytes (SMEs) adhesive characteristics and

irreversible sickle cells (ISCs). The adhesion dynamics of SMEs and ISCs with pulsatile flow under controlled hypoxic condition with increasing wall shear stress was investigated until the detachment of the onset cell. Simulations were carried out for SMEs and ISCs under shear influence with two metrics introduced for quantifying the adhesive process. It was found out that the adhesion of the same cell spring constant increased the shear modulus to initial cell detachment from the functional surface.

Ahmed et al., [24] used partial differential equations (PDE) to form different shapes of red blood cells with fourth order elliptic PDE with boundary conditions applied to form the shape. Different surfaces as a result of the boundary conditions with few parameters were generated with the shape modification of the Red Blood cell using partial differential equation method to design the normal RBC shape. This was capable of creating a parametric surface that is smooth for a blood cell shape as well as complex geometric surface which could easily be modified.

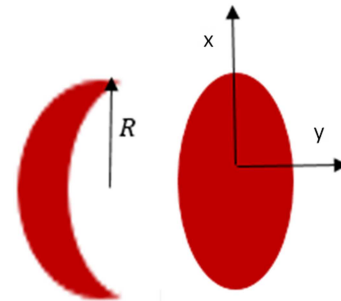
Blood flow and blood disease such as sickle cell anaemia can lead to a host of complications, such as stroke, acute chest syndrome, Pulmonary hypertension, Organ damage, blindness, leg ulcer, gallstone and pregnancy complications. Hence, what are the factors that lead to deformation of normal, biconcave cells to sickle crescent shaped cells, will the treatment model help prevent the sickling of red blood cells or restore memory shape. The expected results will graphically show the influence of pressure, chemical reaction and Energy source which play a significant role in reducing the vaso-occlusion crises associated with patients suffering from sickle cell disease.

## 2. Formulation of the Problem

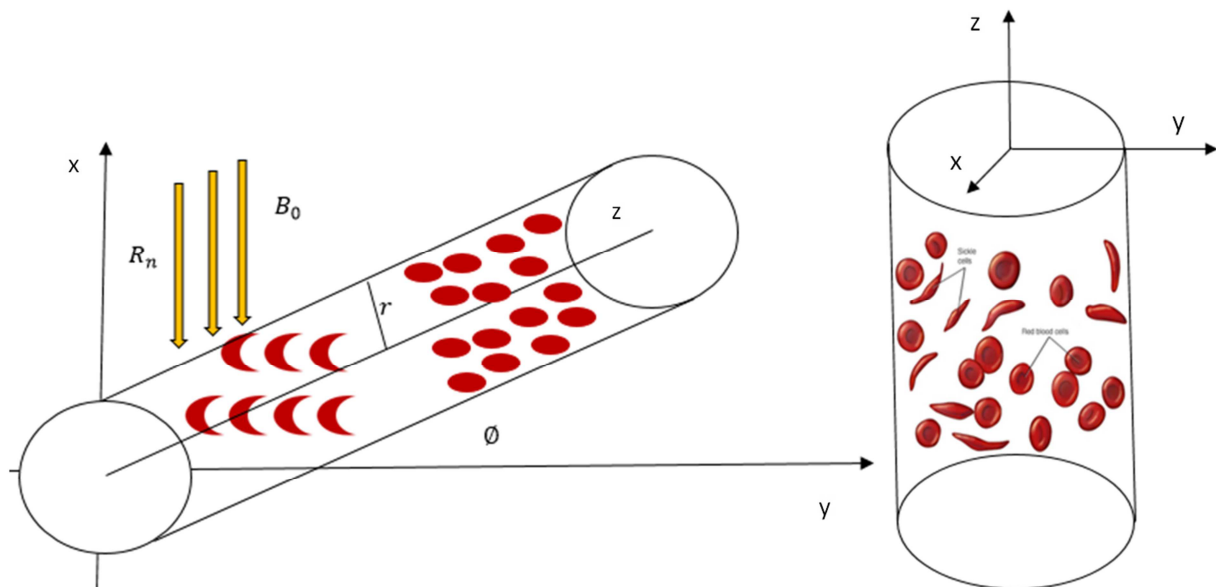
Morphological factors and forces acting in normal and sickled erythrocytes in Sickle cell Anaemia: A Mathematical Model is considered in this study. The dimensional governing equation is transformed to dimensionless governing equation with the analytical solutions of the governing equations expressed in Bessel form.

The mathematical formulation will consider the following assumption such as:

- 1) The Blood is assumed to be a non-Newtonian fluid;
- 2) The fluid motion is both laminar and unsteady;
- 3) Electromagnetic and Buoyance force (interactive force of the fluid membrane) acts on the cell to restore the shape;
- 4) The RBC flow is taken along the  $z$  axis;
- 5) Treatment of sickle cells is considered in the model;
- 6) The model shall be a skeletal framework for the study.



**Figure 3.** Diagram showing morphological effect on the boundary of the sickled RBC.



**Figure 4.** Flow Geometry of sickle RBC restored to normal RBC in the artery.

## 3. Governing Equation

Navier-Stokes equation for the blood flowing axially with the magnetic field, thermal radiation and heat source applied

perpendicularly to the section of occlusion. The blood is assumed to be non-Newtonian with some parameters having an effect on the sickled RBC to restore its normalcy and improve blood flow.

The introduction of heat source, thermal radiation,

buoyance force and chemical reaction/mass diffusion on the model. The temperature and diffusion difference of the blood sickled RBC flowing through the artery is included in the occurred with heat and mass transfer present.

$$\rho \frac{\partial u'}{\partial t'} = -\frac{\partial p'}{\partial z'} + \frac{\mu}{r'} \frac{\partial}{\partial r'} \left( r' \frac{\partial u'}{\partial r'} \right) - \sigma_c B_0^2 u' + g \sin \emptyset - \frac{\mu}{k_p} u' + \rho g B_T (T' - T_0) + \rho g B_C (C' - C_0) \quad (1)$$

$$\frac{\rho C_p}{k_p} \left[ \frac{\partial T'}{\partial t'} \right] = \frac{\partial^2 T'}{\partial r'^2} + \frac{1}{r'} \frac{\partial T'}{\partial r'} + \frac{\rho C_p}{k_p} H_0 T' - \frac{\rho C_p}{k_p} \frac{\partial q_r'}{\partial r'} \quad (2)$$

$$\vartheta \frac{\partial C'}{\partial t'} = \vartheta D' \left[ \frac{\partial^2 C'}{\partial r'^2} + \frac{1}{r'} \frac{\partial C'}{\partial r'} \right] - N' (C' - C_0) \quad (3)$$

The radiative heat flux  $q_r$  is expressed as

$$\frac{\partial q_r'}{\partial r'} = 4(T' - T_0') I' \quad (4)$$

$$I' = \int_0^\infty K_{\lambda f} \frac{de_{b\lambda}}{dT'} d\lambda \quad (5)$$

$I'$  is the absorption coefficient with the plank constant expressed as  $e_{b\lambda}$ .

The pressure gradient and shape morphology is expressed as

$$-\frac{\partial p'}{\partial z'} = P'_s + P'_p e^{i\omega t}; t \geq 0 \quad (6)$$

$$a = \frac{e^{\frac{x^2+y^2}{2R}}}{2\pi R} \quad (7)$$

The boundary slip conditions are

$$\left\{ \begin{array}{l} u' = U'_w, T' = T'_w, C' = C'_w \text{ at } r' = R'(z) \\ u' \rightarrow \infty, T' \rightarrow \infty, C' \rightarrow \infty \text{ at } r' \rightarrow \infty \end{array} \right\} \quad (8)$$

The dimensionless variable introduced in order to write the governing equations and boundary conditions in dimensionless form.

$$\begin{aligned} U'_w = u_0 e^a; T'_w = T_0 + b e^a; T_0 = -c e^a, C'_w = C_0 + d e^a; C_0 = -f e^a, u = \frac{u'}{u_0 a}; u_a = \frac{U'_a}{u_0} r = \frac{r'}{R'_0}; z = \frac{z'}{R'_0}; t = u_0 t'; P = \frac{R'_0 \rho'}{u_0 \mu}; Re = \frac{\rho R_0'^2}{a u_0 \mu}; \theta = \frac{T' - T_0}{T'_w - T_0}; C = \frac{C' - C_0}{C'_w - C_0}; \delta = \frac{\delta'}{R'_0}; H_1 = H_0 R_0'^2; Rn = 4 I' R_0'^2; Mf^2 = \frac{\sigma R_0'^2 a^2 B_0^2}{\mu}; Pr = \frac{R_0'^2 \rho C_p}{u_0 k}; S_c = \frac{\vartheta}{D'}; Cr = \frac{N' R_0'^2}{\vartheta D'}; G_r = \frac{a g \rho R_0'^2 \beta_T \theta (T_w - T_0)}{u_0 \mu}; G_c = \frac{a g \rho R_0'^2 \beta_C (C_w - C_0)}{u_0 \mu}; P_p = \frac{P'_p R_0'^2 a}{u_0 \mu}; P_s = \frac{P'_s R_0'^2 a}{u_0 \mu}; G_0 = \frac{\rho G'_0 R_0'^2}{u_0 \mu}; f_r = \frac{u_0 \mu}{g R_0'^2}; D = \frac{D'}{D_0}; k = \frac{k_p}{a R_0'^2}; \end{aligned} \quad (9)$$

The Dimensional Mathematical Model

$$Re \frac{\partial u}{\partial t} = P_s + P_p e^{i\omega t} + \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) - \left( Mf^2 + \frac{1}{K} \right) u + \frac{\sin \emptyset}{Fr} + G_r \theta + G_c C \quad (10)$$

$$Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} - (H_1 + Rn) \theta \quad (11)$$

$$ScRe \frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} - CrC \quad (12)$$

The boundary conditions are

$$\left\{ \begin{array}{l} u = \frac{e^a}{a}, \theta = 1 - V, C = 1 - W \text{ at } r = a(z) \\ u \rightarrow \infty, \theta \rightarrow \infty, C \rightarrow \infty \text{ at } r \rightarrow \infty \end{array} \right\} \quad (13)$$

## 4. Method of Solution

The partial differential governing equation is solved analytically, with the solutions gotten in Bessel function as Frobenius series for blood velocity, temperature and mass diffusion, expressed as.

### 4.1. Solution to the Blood Flow, Energy and Diffusion

$$u(r, t) = u_0(r) + u_p(r) e^{i\omega t} \quad (14)$$

$$\theta(r, t) = \theta_0(r) + \theta_p(r)\varepsilon e^{i\omega t} \quad (15)$$

$$C(r, t) = C_0(r) + C_p(r)\varepsilon e^{i\omega t} \quad (16)$$

#### 4.2. Bessel Represented in Series

$$J_0 = 1 - \frac{1}{1!} \left(\frac{x}{2}\right)^2 + \frac{1}{2!} \left(\frac{x}{2}\right)^4 - \frac{1}{3!} \left(\frac{x}{2}\right)^6 + \frac{1}{4!} \left(\frac{x}{2}\right)^8 + \dots \quad (17)$$

$$iJ_0 = 1 + \frac{1}{1!} \left(\frac{x}{2}\right)^2 + \frac{1}{2!} \left(\frac{x}{2}\right)^4 + \frac{1}{3!} \left(\frac{x}{2}\right)^6 + \frac{1}{4!} \left(\frac{x}{2}\right)^8 + \dots \quad (18)$$

$$Y_0 = \frac{x}{2} \left( 1 - \frac{1}{1!2!} \left(\frac{x}{2}\right)^2 + \frac{1}{2!3!} \left(\frac{x}{2}\right)^4 - \frac{1}{3!4!} \left(\frac{x}{2}\right)^6 + \dots \right) \quad (19)$$

#### 4.3. Frobenius Series Solution

$$A = \sum_{r=0}^{\infty} a_r x^{k+r} \quad (20)$$

Where  $a_n, k \in u_0, u_1, \theta_0, \theta_1, C_0, C_1$ .

### 5. Solution to the Governing Equation

Substitute equation (16) into equation (12) to obtain the temperature equation for the steady state and pulsatile state.

#### 5.1. Steady State Temperature

$$\frac{\partial^2 \theta_0}{\partial r^2} + \frac{1}{r} \frac{\partial \theta_0}{\partial r} - H\theta_0 = 0 \quad (21)$$

$$H = (H_1 + Rn)$$

#### 5.2. Pulsatile State Temperature

$$\frac{\partial^2 \theta_p}{\partial r^2} + \frac{1}{r} \frac{\partial \theta_p}{\partial r} - \alpha \theta_p = 0 \quad (22)$$

Where  $\alpha = H_1 + Rn - i\omega Pr$ .

The boundary conditions from equation (13) is applied to equation (21) and (22).

The complementary and particular solution for the steady state temperature is expressed as

$$\left\{ \begin{array}{l} \theta_{0h}(r) = A_1 J_0(i\sqrt{H} r) + C_1 Y_0(-i\sqrt{H} r) \\ \theta_{0p}(r) = B_1 + B_2 r \end{array} \right\} \quad (23)$$

The expression  $\theta_{0h}(r)$  in equation (23) is gotten from Equation (21).

At  $r = 0$ , there is regular singularity; hence equation (21) can be solved analytically using the Frobenius series.

The Bessel equation from equation (21) is expressed as

$$\frac{\partial^2 \theta_{0h}}{\partial r^2} + \frac{1}{r} \frac{\partial \theta_{0h}}{\partial r} - H\theta_{0h} = 0 \quad (24)$$

The series solution which form the Funch's theorem is called the Frobenius series expressed as

$$\theta_{0h} = \sum_{r=0}^{\infty} a_r x^{k+r} \quad (25)$$

$$\theta_{0h}' = \sum_{r=0}^{\infty} (k+r) a_r x^{k+r-1} \quad (26)$$

$$\theta_{0h}'' = \sum_{r=0}^{\infty} (k+r)(k+r-1) a_r x^{k+r-2} \quad (27)$$

Substituting  $\theta_{0h}, \theta_{0h}', \theta_{0h}''$  into equation (25)

$$\sum_{r=0}^{\infty} (k+r)(k+r-1) a_r x^{k+r-2} + \sum_{r=0}^{\infty} (k+r) a_r x^{k+r-2} + H \sum_{r=0}^{\infty} a_r x^{k+r} = 0 \quad (28)$$

$$\sum_{r=0}^{\infty} \{[(k+r)(k+r-1) + k+r] x^{k+r-2} + H x^{k+r}\} a_r = 0 \quad (29)$$

Equating the powers of  $x = 0$ , at  $r = 0$ , with the power  $k - 2$ , Then

$$\theta_{0h} = a_0 \left[ 1 + \frac{Hr^2}{(2)^2} + \frac{H^2r^4}{(2)^2(4)^2} + \frac{H^3r^6}{(2)^2(4)^2(6)^2} + \dots \right] \quad (30)$$

Where  $a_0 = 1$ , the expression for the solution of the homogeneous steady state temperature in equation (31) is shown. Hence the solution of  $\theta_{0h}$  is justified as:

$$\theta_{0h}(r) = A_1 J_0(i\sqrt{H}r) + C_1 Y_0(-i\sqrt{H}r)$$

The constant  $C_1 = 0$ , when  $J_0(0) = 1$  and  $Y_0(0) \rightarrow \infty$  since the temperature cannot be infinite at  $r = 0$ . The general solution for  $\theta_0 = \theta_{0h} + \theta_{0p}$ ,

$$\theta_0(r) = A_1 J_0(i\sqrt{H}r) = A_1 \left[ 1 + \frac{Hr^2}{2^2} + \frac{H^2r^4}{2^2 4^2} + \frac{H^3r^6}{2^2 4^2 6^2} + \frac{H^4r^8}{2^2 4^2 6^2 8^2} + \dots \right] \quad (31)$$

Applying the boundary condition in equation (13)

Where

$$A_1 = \left( \frac{1-V}{1 + \frac{Ha^2}{2^2} + \frac{H^2a^4}{2^2 4^2} + \frac{H^3a^6}{2^2 4^2 6^2} + \frac{H^4a^8}{2^2 4^2 6^2 8^2} + \dots} \right) \quad (32)$$

Where

$$J_0(i\sqrt{N}r) = \left[ 1 + \frac{Hr^2}{2^2} + \frac{H^2r^4}{2^2 4^2} - \frac{H^3r^6}{2^2 4^2 6^2} + \frac{H^4r^8}{2^2 4^2 6^2 8^2} + \dots \right] \quad (33)$$

Substitute equation (32) into equation (31) the solution for the steady state temperature is expressed as

$$\theta_0(r) = \frac{(1-V) \left[ 1 + \frac{Hr^2}{2^2} + \frac{H^2r^4}{2^2 4^2} + \frac{H^3r^6}{2^2 4^2 6^2} + \frac{H^4r^8}{2^2 4^2 6^2 8^2} + \dots \right]}{\left[ 1 + \frac{Ha^2}{2^2} + \frac{H^2a^4}{2^2 4^2} + \frac{H^3a^6}{2^2 4^2 6^2} + \frac{H^4a^8}{2^2 4^2 6^2 8^2} + \dots \right]} \quad (34)$$

The pulsatile state homogeneous solution for the temperature is gotten by applying the same method for the steady state with the particular solution for the pulsatile state velocity blood flow expressed below as

$$\theta_{ph} = a_0 \left[ 1 + \frac{\alpha r^2}{(2)^2} + \frac{\alpha^2 r^4}{(2)^2(4)^2} + \frac{\alpha^3 r^6}{(2)^2(4)^2(6)^2} + \dots \right] \quad (35)$$

Where  $a_0 = 1$  and  $\alpha = H^2 - i\omega Pr$ , Hence the solution of  $\theta_{ph}$  is justified as:

$$\left\{ \begin{array}{l} \theta_{ph}(r) = A_2 J_0(i\sqrt{\alpha}r) + C_2 Y_0(-i\sqrt{\alpha}r) \\ \theta_{pp}(r) = B_1 + B_2 r \end{array} \right\} \quad (36)$$

At  $r = 0$ , there is regular singularity, hence equation (22) can be solved analytically using the Frobenius series

The constant  $C_2 = 0$ , when  $J_0(0) = 1$  and  $Y_0(0) \rightarrow \infty$  since the temperature cannot be infinite at  $r = 0$ . The general solution for  $\theta_p = \theta_{ph} + \theta_{pp}$ , where  $B_2 = 0$  and  $B_1 = 0$ , will be expressed as

$$\theta_p(r) = A_2 J_0(i\sqrt{\alpha}r) = A_2 \left[ 1 + \frac{\alpha r^2}{2^2} + \frac{\alpha^2 r^4}{2^2 4^2} + \frac{\alpha^3 r^6}{2^2 4^2 6^2} + \frac{\alpha^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right] \quad (37)$$

Applying the boundary condition in equation (13)

Where

$$J_0(i\sqrt{\alpha}R) = \left[ 1 + \frac{\alpha r^2}{2^2} + \frac{\alpha^2 r^4}{2^2 4^2} + \frac{\alpha^3 r^6}{2^2 4^2 6^2} + \frac{\alpha^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right] \quad (38)$$

$$\theta_p(r) = \frac{(1-V) \left[ 1 + \frac{\alpha r^2}{2^2} + \frac{\alpha^2 r^4}{2^2 4^2} + \frac{\alpha^3 r^6}{2^2 4^2 6^2} + \frac{\alpha^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right]}{\left[ 1 + \frac{\alpha a^2}{2^2} + \frac{\alpha^2 a^4}{2^2 4^2} + \frac{\alpha^3 a^6}{2^2 4^2 6^2} + \frac{\alpha^4 a^8}{2^2 4^2 6^2 8^2} + \dots \right]} \quad (39)$$

The temperature is gotten by substituting equation (34) and equation (39) into equation (15)

$$\theta(r, t) = \frac{(1-V) \left[ 1 + \frac{Hr^2}{2^2} + \frac{H^2r^4}{2^2 4^2} + \frac{H^3r^6}{2^2 4^2 6^2} + \frac{H^4r^8}{2^2 4^2 6^2 8^2} + \dots \right]}{\left[ 1 + \frac{Ha^2}{2^2} + \frac{H^2a^4}{2^2 4^2} + \frac{H^3a^6}{2^2 4^2 6^2} + \frac{H^4a^8}{2^2 4^2 6^2 8^2} + \dots \right]} + \left( \frac{(1-V) \left[ 1 + \frac{\alpha r^2}{2^2} + \frac{\alpha^2 r^4}{2^2 4^2} + \frac{\alpha^3 r^6}{2^2 4^2 6^2} + \frac{\alpha^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right]}{\left[ 1 + \frac{\alpha a^2}{2^2} + \frac{\alpha^2 a^4}{2^2 4^2} + \frac{\alpha^3 a^6}{2^2 4^2 6^2} + \frac{\alpha^4 a^8}{2^2 4^2 6^2 8^2} + \dots \right]} \right) \epsilon e^{i\omega t} \quad (40)$$

### 5.3. Steady State Concentration

$$\frac{\partial^2 C_0}{\partial r^2} + \frac{1}{r} \frac{\partial C_0}{\partial r} - CrC_0 = 0 \quad (41)$$

### 5.4. Pulsatile State Concentration

$$\frac{\partial^2 C_p}{\partial r^2} + \frac{1}{r} \frac{\partial C_p}{\partial r} - \alpha_2 C_p = 0 \quad (42)$$

Where  $\alpha_2 = Cr + i\omega ScRe$ .

The boundary conditions in equation (13) is applied to equation (41) and (42).

The complementary and particular solution for the steady state concentration is expressed as

$$\begin{cases} C_{0h}(r) = A_3 J_0(i\sqrt{Cr} r) + C_3 Y_0(-i\sqrt{Cr} r) \\ C_{0p}(r) = B_1 + B_2 r \end{cases} \quad (43)$$

The constant  $C_3 = 0$ , when  $J_0(0) = 1$  and  $Y_0(0) \rightarrow \infty$  since the concentration cannot be infinite at  $r = 0$ , the general solution for  $C_0 = C_{0h} + C_{0p}$ , where  $B_2 = 0$  and  $B_1 = 0$  is

$$C_0(r) = A_3 J_0(i\sqrt{Cr} r) = A_3 \left[ 1 + \frac{Cr r^2}{2^2} + \frac{Cr^2 r^4}{2^2 4^2} + \frac{Cr^3 r^6}{2^2 4^2 6^2} + \frac{Cr^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right] \quad (44)$$

Where

$$J_0(i\sqrt{Cr} r) = \left[ 1 + \frac{Cr r^2}{2^2} + \frac{Cr^2 r^4}{2^2 4^2} + \frac{Cr^3 r^6}{2^2 4^2 6^2} + \frac{Cr^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right] \quad (45)$$

The steady state concentration is expressed as

$$C_0(r) = \frac{(1-W) \left[ 1 + \frac{Cr r^2}{2^2} + \frac{Cr^2 r^4}{2^2 4^2} + \frac{Cr^3 r^6}{2^2 4^2 6^2} + \frac{Cr^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right]}{\left[ 1 + \frac{Cr a^2}{2^2} + \frac{Cr^2 a^4}{2^2 4^2} + \frac{Cr^3 a^6}{2^2 4^2 6^2} + \frac{Cr^4 a^8}{2^2 4^2 6^2 8^2} + \dots \right]} \quad (46)$$

The complementary and particular solution for the pulsatile state temperature is expressed as

$$\begin{cases} C_{ph}(r) = A_4 J_0(i\sqrt{\alpha_2} r) + C_4 I_0(-i\sqrt{\alpha_2} r) \\ C_{pp}(r) = B_1 + B_2 r \end{cases} \quad (47)$$

The constant  $C_4 = 0$ , when  $J_0(0) = 1$  and  $Y_0(0) \rightarrow \infty$  since the concentration cannot be infinite at  $r = 0$ , the general solution for  $C_p = C_{ph} + C_{pp}$ , where  $B_2 = 0$  and  $B_1 = 0$ , will be expressed as

$$C_p(r) = A_4 J_0(i\sqrt{\alpha_2} r) = A_4 \left[ 1 + \frac{\alpha_2 r^2}{2^2} + \frac{\alpha_2^2 r^4}{2^2 4^2} + \frac{\alpha_2^3 r^6}{2^2 4^2 6^2} + \frac{\alpha_2^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right] \quad (48)$$

$$\text{Where } J_0(i\sqrt{\alpha_2} r) = \left[ 1 + \frac{\alpha_2 r^2}{2^2} + \frac{\alpha_2^2 r^4}{2^2 4^2} + \frac{\alpha_2^3 r^6}{2^2 4^2 6^2} + \frac{\alpha_2^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right] \quad (49)$$

The pulsatile state concentration is expressed as

$$C_p(r) = \frac{(1-W) \left[ 1 + \frac{\alpha_2 r^2}{2^2} + \frac{\alpha_2^2 r^4}{2^2 4^2} + \frac{\alpha_2^3 r^6}{2^2 4^2 6^2} + \frac{\alpha_2^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right]}{\left[ 1 + \frac{\alpha_2 a^2}{2^2} + \frac{\alpha_2^2 a^4}{2^2 4^2} + \frac{\alpha_2^3 a^6}{2^2 4^2 6^2} + \frac{\alpha_2^4 a^8}{2^2 4^2 6^2 8^2} + \dots \right]} \quad (50)$$

The concentration is gotten by substituting equation (46) and equation (50) into equation (16)

$$C(r, t) = \frac{(1-W) \left[ 1 + \frac{Cr r^2}{2^2} + \frac{Cr^2 r^4}{2^2 4^2} + \frac{Cr^3 r^6}{2^2 4^2 6^2} + \frac{Cr^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right]}{\left[ 1 + \frac{Cr a^2}{2^2} + \frac{Cr^2 a^4}{2^2 4^2} + \frac{Cr^3 a^6}{2^2 4^2 6^2} + \frac{Cr^4 a^8}{2^2 4^2 6^2 8^2} + \dots \right]} + \left( \frac{(1-W) \left[ 1 + \frac{\alpha_2 r^2}{2^2} + \frac{\alpha_2^2 r^4}{2^2 4^2} + \frac{\alpha_2^3 r^6}{2^2 4^2 6^2} + \frac{\alpha_2^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right]}{\left[ 1 + \frac{\alpha_2 a^2}{2^2} + \frac{\alpha_2^2 a^4}{2^2 4^2} + \frac{\alpha_2^3 a^6}{2^2 4^2 6^2} + \frac{\alpha_2^4 a^8}{2^2 4^2 6^2 8^2} + \dots \right]} \right) \epsilon e^{i\omega t} \quad (51)$$

Substitute equation (40) and (51) into equation (14), to obtain the flow equation for the steady state and pulsatile state.

### 5.5. Steady State Velocity Blood Flow

$$\frac{\partial^2 u_0}{\partial r^2} + \frac{1}{r} \frac{\partial u_0}{\partial r} - \beta_1 u_0 = -G - \frac{(1-V) \left[ 1 + \frac{Hr^2}{2^2} + \frac{H^2 r^4}{2^2 4^2} + \frac{H^3 r^6}{2^2 4^2 6^2} + \frac{H^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right]}{\left[ 1 + \frac{Ha^2}{2^2} + \frac{H^2 a^4}{2^2 4^2} + \frac{H^3 a^6}{2^2 4^2 6^2} + \frac{H^4 a^8}{2^2 4^2 6^2 8^2} + \dots \right]} - \frac{(1-W) \left[ 1 + \frac{Cr r^2}{2^2} + \frac{Cr^2 r^4}{2^2 4^2} + \frac{Cr^3 r^6}{2^2 4^2 6^2} + \frac{Cr^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right]}{\left[ 1 + \frac{Cr a^2}{2^2} + \frac{Cr^2 a^4}{2^2 4^2} + \frac{Cr^3 a^6}{2^2 4^2 6^2} + \frac{Cr^4 a^8}{2^2 4^2 6^2 8^2} + \dots \right]} \quad (52)$$

Where  $\beta_1 = Mf^2 + \frac{1}{K}$  and  $G = P_s + \frac{\sin \phi}{Fr}$ .

### 5.6. Pulsatile State Velocity Blood Flow

$$\frac{\partial^2 u_p}{\partial r^2} + \frac{1}{r} \frac{\partial u_p}{\partial r} - \beta_2 u_p = -P_p - \frac{(1-V) \left[ 1 + \frac{\alpha r^2}{2^2} + \frac{\alpha^2 r^4}{2^2 4^2} + \frac{\alpha^3 r^6}{2^2 4^2 6^2} + \frac{\alpha^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right]}{\left[ 1 + \frac{\alpha a^2}{2^2} + \frac{\alpha^2 a^4}{2^2 4^2} + \frac{\alpha^3 a^6}{2^2 4^2 6^2} + \frac{\alpha^4 a^8}{2^2 4^2 6^2 8^2} + \dots \right]} - \frac{(1-W) \left[ 1 + \frac{\alpha_2 r^2}{2^2} + \frac{\alpha_2^2 r^4}{2^2 4^2} + \frac{\alpha_2^3 r^6}{2^2 4^2 6^2} + \frac{\alpha_2^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right]}{\left[ 1 + \frac{\alpha_2 a^2}{2^2} + \frac{\alpha_2^2 a^4}{2^2 4^2} + \frac{\alpha_2^3 a^6}{2^2 4^2 6^2} + \frac{\alpha_2^4 a^8}{2^2 4^2 6^2 8^2} + \dots \right]} \quad (53)$$

Where  $\beta_2 = Mf^2 + \frac{1}{K} + Rei\omega$ .

The boundary condition in equation (13) is applied to equation (52) and (53).

The complementary and particular solution for the steady state velocity blood flow is

$$\begin{cases} u_{0h}(r) = A_5 J_0(i\sqrt{\beta_1}r) + C_5 Y_0(-i\sqrt{\beta_1}r) \\ u_{0p}(r) = G_0 + G_1 r^2 + G_2 r^4 + G_3 r^6 + G_4 r^8 \end{cases} \quad (54)$$

The constant  $C_5 = 0$ , when  $J_0(0) = 1$  and  $Y_0(0) \rightarrow \infty$  since the velocity cannot be infinite at  $r = 0$ , the general solution for  $u_0 = u_{0h} + u_{0p}$ ,

$$u_0(r) = A_5 J_0(i\sqrt{\beta_1}r) + G_0 + G_1 r^2 + G_2 r^4 + G_3 r^6 + G_4 r^8 \quad (55)$$

Where

$$J_0(i\sqrt{\beta_1}R) = \left[ 1 + \frac{\beta_1 r^2}{2^2} + \frac{\beta_1^2 r^4}{2^2 4^2} + \frac{\beta_1^3 r^6}{2^2 4^2 6^2} + \frac{\beta_1^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right] \quad (56)$$

The steady state velocity blood flow equation expression will be

$$u_0(r) = A_5 [J_0(i\sqrt{\beta_1}R)] + G_0 + G_1 r^2 + G_2 r^4 + G_3 r^6 + G_4 r^8 \quad (57)$$

The complementary and particular solution for the pulsatile state velocity blood flow is

$$\begin{cases} u_{ph}(r) = A_6 J_0(i\sqrt{\beta_2}r) + C_6 I_0(-i\sqrt{\beta_2}r) \\ u_{pp}(r) = H_0 + H_1 r^2 + H_2 r^4 + H_3 r^6 + H_4 r^8 \end{cases} \quad (58)$$

$J_0(0) = 1$  and  $Y_0(0) \rightarrow \infty$ , the constant  $C_6 = 0$ , since the velocity cannot be infinite at  $r = 0$ . The general solution for  $u_p = u_{ph} + u_{pp}$ ,

$$u_p(r) = A_6 J_0(i\sqrt{\beta_2}r) + H_0 + H_1 r^2 + H_2 r^4 + H_3 r^6 + H_4 r^8 \quad (59)$$

Where

$$J_0(i\sqrt{\beta_2}R) = \left[ 1 + \frac{\beta_2 r^2}{2^2} + \frac{\beta_2^2 r^4}{2^2 4^2} + \frac{\beta_2^3 r^6}{2^2 4^2 6^2} + \frac{\beta_2^4 r^8}{2^2 4^2 6^2 8^2} + \dots \right] \quad (60)$$

$$u_p(r) = u_{pc}(r) + u_{pp}(r).$$

$$u_p = A_6 [J_0(i\sqrt{\beta_2}R)] + H_0 + H_1 r^2 + H_2 r^4 + H_3 r^6 + H_4 r^8 \quad (61)$$

The general solution for the velocity blood flow will be expressed as

$$u(r, t) = A_5 [J_0(i\sqrt{\beta_1}R)] + G_0 + G_1 r^2 + G_2 r^4 + G_3 r^6 + G_4 r^8 + \{A_6 [J_0(i\sqrt{\beta_2}R)] + H_0 + H_1 r^2 + H_2 r^4 + H_3 r^6 + H_4 r^8\} \epsilon e^{i\omega t} \quad (62)$$

An alternate analytical solution to the velocity flow with restored RBC is

$$u(r, t) = \left[ \frac{\frac{e^a}{a}}{J_0(i\sqrt{\beta_1}a)} - \frac{P_0 + \frac{\sin \phi}{Fr}}{\beta_1 J_0(i\sqrt{\beta_1}a)} - \frac{G_r \left( \frac{J_0(i\sqrt{Na})}{J_0(i\sqrt{\beta_1}a)} \right) - \frac{G_c \left( \frac{J_0(i\sqrt{Kr}a)}{J_0(i\sqrt{\beta_1}a)} \right)}{\beta_1} \right] J_0(i\sqrt{\beta_1}r) + \frac{P_0 + \frac{\sin \phi}{Fr}}{\beta_1} + \frac{G_r \left( \frac{J_0(i\sqrt{Nr})}{J_0(i\sqrt{Na})} \right) + \frac{G_c \left( \frac{J_0(i\sqrt{Kr}r)}{J_0(i\sqrt{Kr}a)} \right)}{\beta_1} + \left\{ \left[ \frac{\frac{e^a}{a}}{J_0(i\sqrt{\beta_2}a)} - \frac{M^2 + \frac{1}{K} + Rei\omega}{\beta_2 J_0(i\sqrt{\beta_2}a)} - \frac{G_r \left( \frac{J_0(i\sqrt{\alpha_1}a)}{J_0(i\sqrt{\beta_2}a)} \right) - \frac{G_c \left( \frac{J_0(i\sqrt{\alpha_2}a)}{J_0(i\sqrt{\beta_2}a)} \right)}{\beta_2} \right] J_0(i\sqrt{\beta_2}r) + \frac{M^2 + \frac{1}{K} + Rei\omega}{\beta_2} + \frac{G_r J_0(\sqrt{\alpha_1}r)}{\beta_2 J_0(i\sqrt{\alpha_1}a)} + \frac{G_c J_0(\sqrt{\alpha_2}r)}{\beta_2 J_0(i\sqrt{\alpha_2}a)} \right\} \epsilon e^{i\omega t} \quad (63)$$



### 5.7. Solution to the Wall Shear Stress

$$\tau(u, t) = \mu \left( \frac{du}{dr} \right) = A_5 \left[ \frac{\beta_1 r}{2} + \frac{\beta_1^2 r^3}{2^2 4} + \frac{\beta_1^3 r^5}{2^2 4^2 6} + \frac{\beta_1^4 r^7}{2^2 4^2 6^2 8} + \dots \right] + 2G_1 r + 4G_2 r^3 + 6G_3 r^5 + 8G_4 r^7 + \left\{ A_6 \left[ \frac{\beta_2 r}{2} + \frac{\beta_2^2 r^3}{2^2 4} + \frac{\beta_2^3 r^5}{2^2 4^2 6} + \frac{\beta_2^4 r^7}{2^2 4^2 6^2 8} + \dots \right] + 2H_1 r + 4H_2 r^3 + 6H_3 r^5 + 8H_4 r^7 \right\} \epsilon e^{i\omega t} \quad (64)$$

An alternate solution to the wall shear stress

$$\tau(u, t) = \mu \left( \frac{du}{dr} \right) = \mu \left\{ \left[ \frac{\frac{e^a}{a}}{J_0(i\sqrt{\beta_1} a)} - \frac{G}{\beta_1 J_0(i\sqrt{\beta_1} a)} - \frac{G_r}{\beta_1} \left( \frac{J_0(i\sqrt{N} a)}{J_0(i\sqrt{\beta_1} a)} \right) - \frac{G_c}{\beta_1} \left( \frac{J_0(i\sqrt{K} a)}{J_0(i\sqrt{\beta_1} a)} \right) \right] i\sqrt{\beta_1} J_1(i\sqrt{\beta_1} r) - i\sqrt{N} \frac{G_r}{\beta_1} \left( \frac{J_1(i\sqrt{N} r)}{J_0(i\sqrt{N} a)} \right) - i\sqrt{K} \frac{G_c}{\beta_1} \left( \frac{J_1(i\sqrt{K} r)}{J_0(i\sqrt{K} a)} \right) - \left[ \frac{\frac{e^a}{a}}{J_0(i\sqrt{\beta_2} a)} - \frac{F}{\beta_2 J_0(i\sqrt{\beta_2} a)} - \frac{G_r}{\beta_2} \left( \frac{J_0(i\sqrt{\alpha_1} a)}{J_0(i\sqrt{\beta_2} a)} \right) - \frac{G_c}{\beta_2} \left( \frac{J_0(i\sqrt{\alpha_2} a)}{J_0(i\sqrt{\beta_2} a)} \right) \right] i\sqrt{\beta_2} J_1(i\sqrt{\beta_2} r) - \frac{G_r \sqrt{\alpha_1} J_0(i\sqrt{\alpha_1} r)}{\beta_2 J_0(i\sqrt{\beta_2} a)} - \frac{G_c \sqrt{\alpha_2} J_0(i\sqrt{\alpha_2} r)}{\beta_2 J_0(i\sqrt{\beta_2} a)} \right\} \epsilon e^{i\omega t} \quad (65)$$

## 6. Graphical Results and Discussion

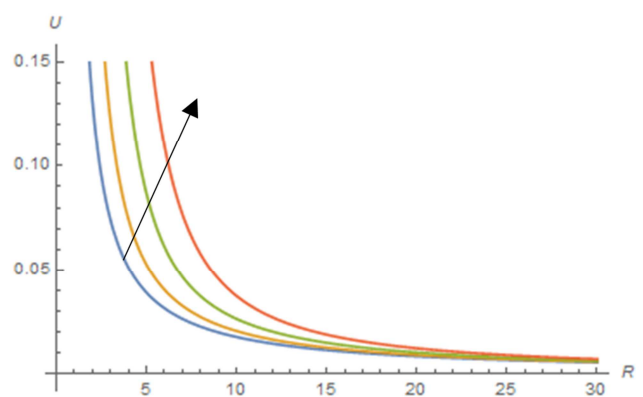
The behavior of some parameters on the blood flow and wall shear stress is shown graphically. These parameters have the effect on the deformed red blood cell (RBC) such that it improves the shape and hence causes an increased flow or vice versa. The deformed RBC is responsible for the sickle cell disease (SCD). Figure 5 shows the morphological effect on the RBC when it takes a sickled shape. Figure 6 shows a fluctuating behavior of the blood flow with varying inclination of the angle from 15° to 30°. This could cause an irregular pattern in the blood flow. Figure 7 shows a decrease in the blood flow due an increase in the magnetic field. The enhanced magnetic field reduced the blood viscosity by introducing a Lorentz force which diminishes the blood flow. Since the blood is sickle in nature, magnetic field increase stiffens the red blood cell (RBC) and creates possible occlusion in the artery. Hence this parameter will not be a healthy treatment measure for the sickle cell patient. Figure 8 shows an increase in the blood flow as the pulsatile pressure increases. A disturbance of the blood flow is created, reducing the blood viscosity and improving the blood flow. Figure 9 shows blood flow increase as the heat source increases reducing the viscosity of the blood and improving the RBC. Figure 10 and Figure 11 shows the increase in the interactive force of the fluid membrane called buoyance force for Grashof's temperature and Grashof's diffusion number causing an increase in the blood flow. This forces applied on the artery with sickle cell disease (SCD), creates a reduced viscous force internally. This could reduce the pains and create easy flow of RBC through the arteries and veins. Figure 12 shows an increase in chemical reaction causing an increase in blood flow decreasing the viscosity of the (RBC) and improving the shape memory resulting. This may be helpful in other health challenges and for patients with (SCD).

Figure 13, shows the change in the behavior of the blood flow with fluctuation in inclined angle from 15° to 30° creating an irregular effect on the shear stress of the artery

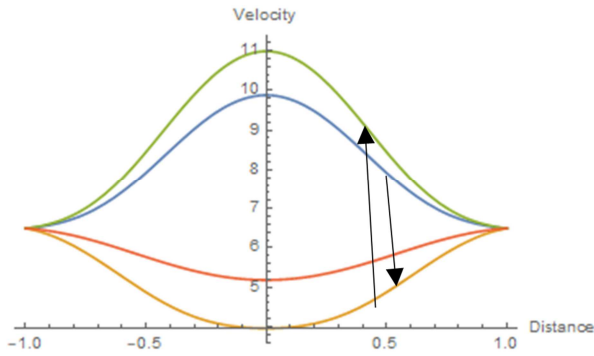
wall. Figure 14 shows an increase in the magnetic field increases the shear stress at the artery wall while Figure 15 and figure 16 shows an increase in the pulsatile pressure and heat source decreases the shear stress at the artery wall. From Figure 17 and Figure 18 shows an increase in the buoyance force for Grashof's temperature and Grashof's diffusion number decreases the shear stress at the artery wall while Figure 19 shows an increase in the chemical reaction increases the shear stress at the artery wall.

**Table 1.** Table showing quantities and values.

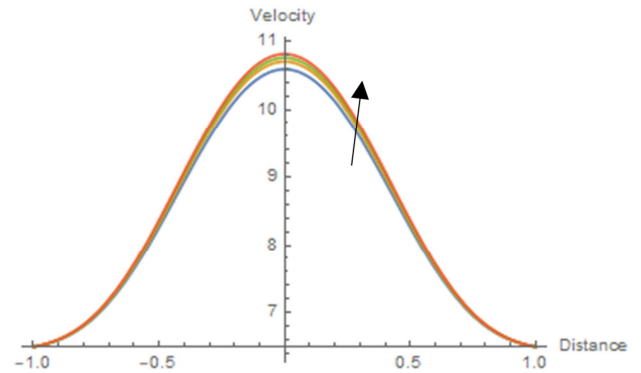
S/N	QUANTITY	VALUES
1	Angle of inclination of artery $\phi$	30°
2	Schmidt number $Sc$	1
3	Chemical reaction $Cr$	1
4	Grashof temperature number $Gr$	2
5	Grashof diffusion number $Gc$	3
6	Heat source $H$	0.5
7	Peclet number $Pe$	1
8	Steady state pressure $Ps$	2
9	Pulsatile state pressure $Pp$	4
10	Froude number $Fr$	0.05
11	Permeability of porous medium $k$	0.1
12	Amplitude of frequency $\omega$	1
13	Magnetic field $Mf$	1.5
14	Time $t$	1
15	Radius of artery $R$	0.55



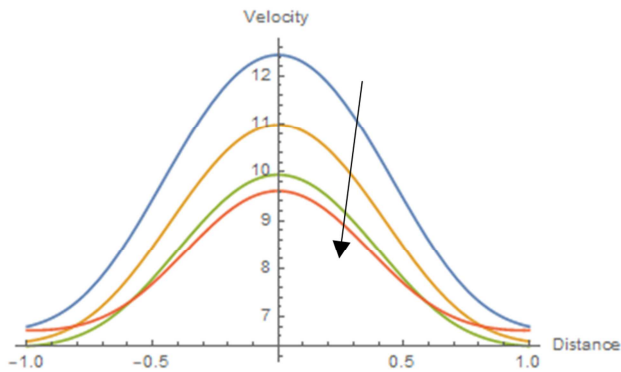
**Figure 5.** Diagram showing morphological changes from sickled RBC to normal RBC.



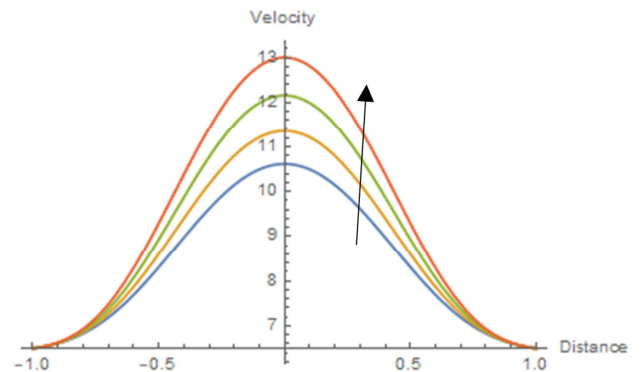
**Figure 6.** Velocity of Blood flow with varying values of inclined artery  $\phi=15^\circ, 30^\circ, 45^\circ, 60^\circ$ .



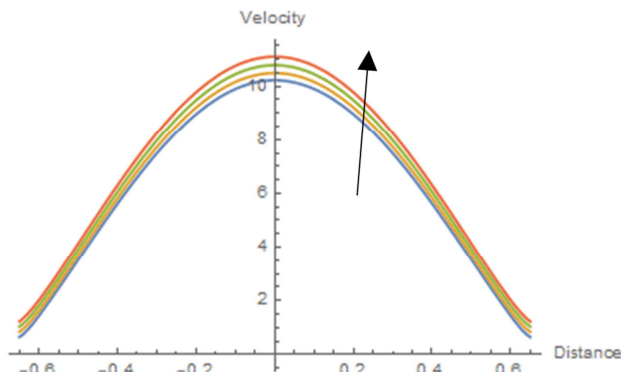
**Figure 10.** Velocity of Blood flow with varying values of Grashof temperature number  $Gr=1, 2, 3, 4$ .



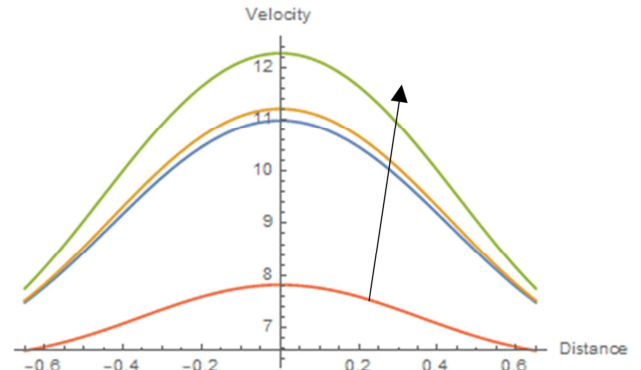
**Figure 7.** Velocity of Blood flow with varying values of Magnetic field  $Mf=0.75, 1, 1.25, 1.5$ .



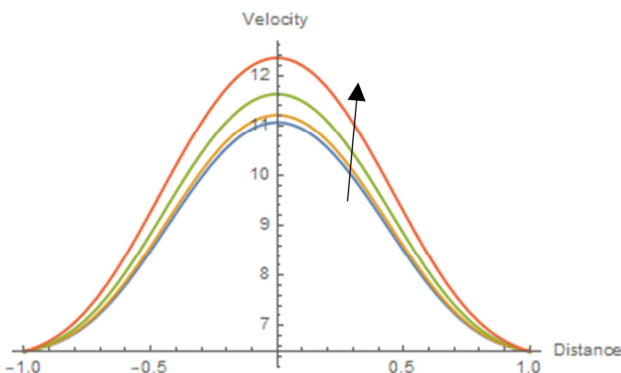
**Figure 11.** Velocity of Blood flow with varying values of Grashof Diffusion number  $Gc=1, 2, 3, 4$ .



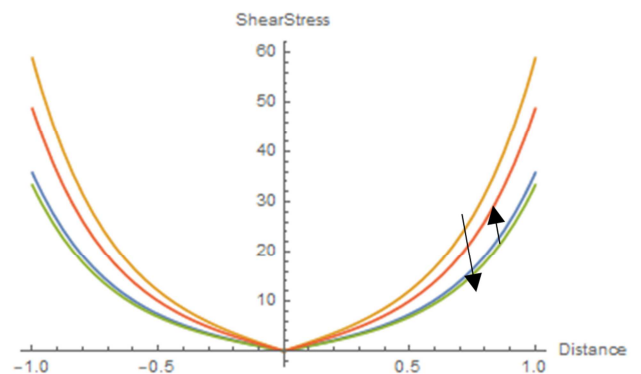
**Figure 8.** Velocity of Blood flow with varying values of pulsatile pressure  $Pp=2, 4, 6, 8$ .



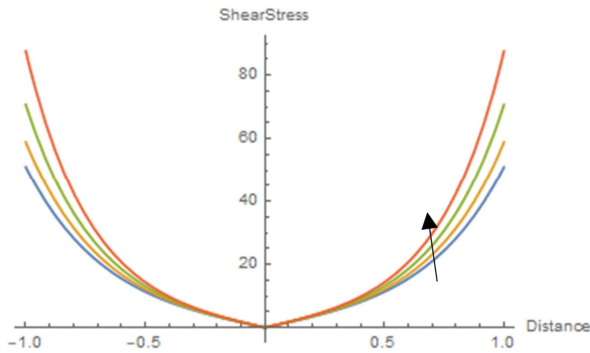
**Figure 12.** Velocity of Blood flow with varying values of Chemical Reaction  $Kr=1, 3, 5, 7$ .



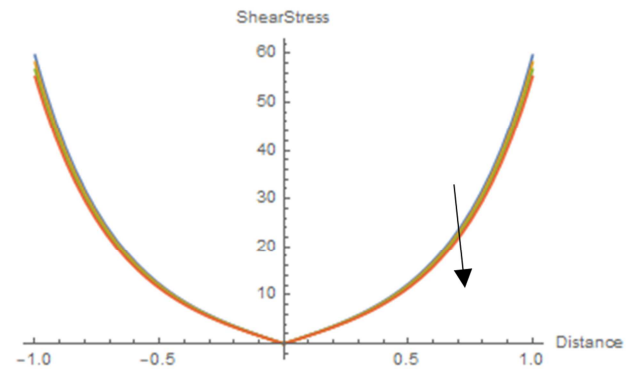
**Figure 9.** Velocity of Blood flow with varying values of Heat Source  $H=0.5, 1, 1.5, 2$ .



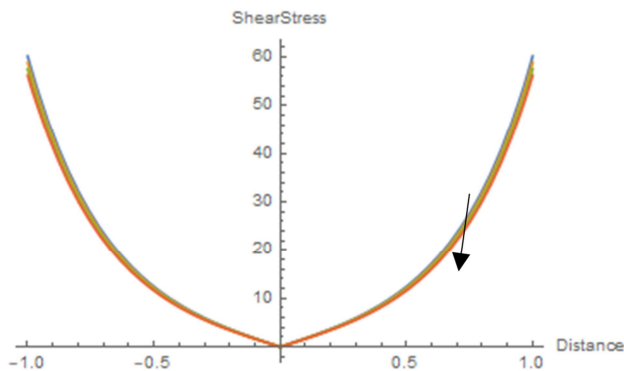
**Figure 13.** Velocity of Blood flow with varying values of inclined artery  $\phi=15^\circ, 30^\circ, 45^\circ, 60^\circ$ .



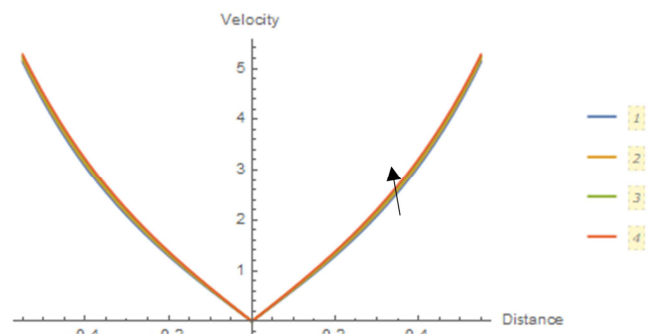
**Figure 14.** Velocity of Blood flow with varying values of Magnetic field  $Mf=0.75, 1, 1.25, 1.5$ .



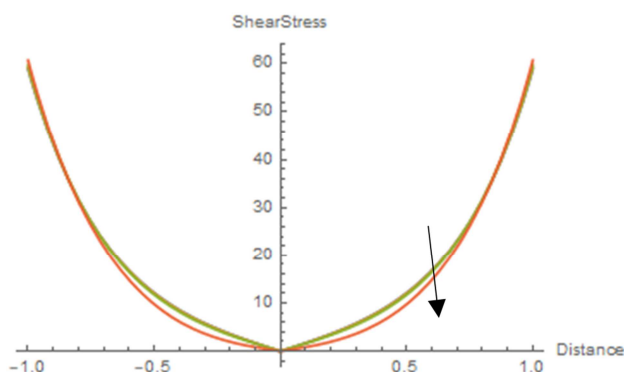
**Figure 18.** Velocity of Blood flow with varying values of Grashof Diffusion number  $Gc=1, 2, 3, 4$ .



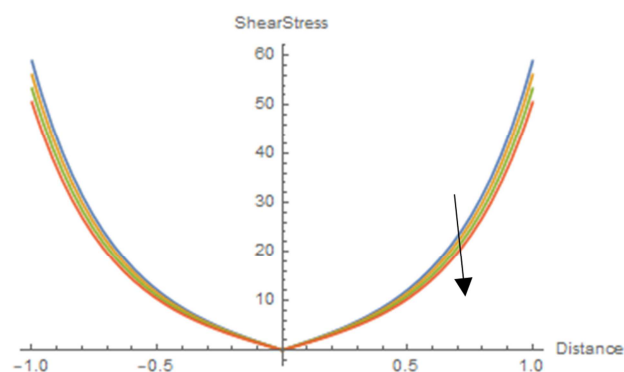
**Figure 15.** Velocity of Blood flow with varying values of Pulsatile pressure  $Pl=2, 4, 6, 8$ .



**Figure 19.** Velocity of Blood flow with varying values of Chemical Reaction  $Cr=1, 3, 5, 7$ .



**Figure 16.** Velocity of Blood flow with varying values of Heat Source  $H=0.5, 1, 1.5, 2$ .



**Figure 17.** Velocity of Blood flow with varying values of Grashof temperature number  $Gr=1, 2, 3, 4$ .

## 7. Conclusion

Certain parameters had an improvement on the deformed RBC while others had no improved effect on the deformed RBC. In conclusion, it was observed that:

- i). The Magnetic field increase, decreases the flow of blood, increases the shear stress at the wall and decreases the volumetric flow rate. No improved effect on the deformed RBC.
- ii). Increase in chemical reaction increased pulsatile pressure which results to an increase in the blood flow, decreases the shear stress at the wall and increases the volumetric flow rate. There is an improved effect on the deformed RBC which will reduce the crises experienced in patients with SCD.
- iii). Heat source increase, increases the flow of blood, reduces the shear stress at the wall and increases the volumetric flow rate. There is an improved effect on the deformed RBC which will reduce the crises experienced in patients with SCD.
- iv). An increase in inclination angle had an irregular pattern on the flow of blood, the shear stress at the wall and volumetric flow rate. There is no significant effect on the deformed RBC and there could be triggering of SCD crises.
- v). Buoyance force increase for Grashof temperature number and Grashof diffusion number, increases the flow of blood, reduces the shear stress at the wall and increases the volumetric flow rate. There is an

improved effect on the deformed RBC which will reduce the crises experienced in patients with SCD.

## Acknowledgements

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