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Application of Log and Exponential Functions for Velocity Calculation in Axis Symmetrical Conductor's Cross-Section

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Abstract: The paper is occupied with laminar and turbulence flow in round conductors having as main theme velocity division in pipes cross – section. For this problem there are existing formulas that are analyzed, and also some new formulas are presented. Rectilinear flow is researched with constant cross – section therefor flow through all cross – sections is parallel and normal on cross – section. Flow is uniform through flow direction, average velocity is constant value. Laminar flow where analytic solution is existed and turbulence flow where experimental formulas are applied, were analyzed. Log and exponential formulas for velocity division in turbulence flow are analyzed through short calculation and the graphic for identical flow conditions with same *Re* number. *Reynolds* equation applied for steady turbulence flow, for flow plane do not allowed determination of velocity division in cross – section. Therefor relations between average values and fluctuations were assumed as log or exponential functions and they were experimentally validated. Some existing formulas are shown below. Laminar uniform axis symmetrical flow having analytic solution as square parable is shown. Formula for turbulence axis symmetrical flow with log velocity division called "velocity deficit" is analyzed. Also formula for turbulence axis symmetrical flow with exponential velocity division for smooth wall has been analyzed. At the end new and original formulas for turbulence flow with log and exponential velocity division are presented having significant advantages. These mathematical formula have to be validated experimentally and justified for use in some areas of fluid mechanics. Analyze is valid also for plane flow between two plane boards.

Keywords: Log, Exponential, Function, Velocity, Division, Original

1. Introduction

The paper will announced usage of log and exponential functions for velocity calculation in cross-section of round conductors.

Rectilinear flow is researched with constant cross-section therefor average flow through all cross-sections is rectilinear and parallel, normal on cross-section. (Mostly turbulence flow is considered.) Flow is uniform through flow direction, average velocity is constant value.

Laminar flow is researched in the round conductors (axis symmetrical flow) in case there was analytic solution for velocity division. Also turbulence flow in round conductors is analyzed where applied `formulas were got by experiments using log and exponential functions.

At the end new and original mathematical formulas are presented which could be useful for research in specific areas of fluid mechanics.

Each formula presented velocity division is analyzed through similar short calculation and the graphic, for same flow conditions. All calculations are derived for turbulence flow with same Re number according to for the same average velocity of water and the same conductor diameter.

Reynolds equation:

$$0 = - \partial (gZ + p/\varrho)/\partial x_1 + \mu/\varrho\, \partial^2\overline{u}_1/\partial x_2{}^2 + \partial(-\overline{u'_1 u'_2})/\partial x_2 \qquad (1)$$

Applied for turbulence steady flow for plane flow do not allowed determination of velocity division in cross-section $\overline{u}_1 = \overline{u}_1(x_2)$. Relations bitween average values and fluctuations in turbulence flow have to somehow be determinated, thus this relations were assumed and were accepted if they had experimental confirmation. [1-4].

This way a lot of log and exponential formulas were derived, having experimentaly validation and some of them are analyzed.

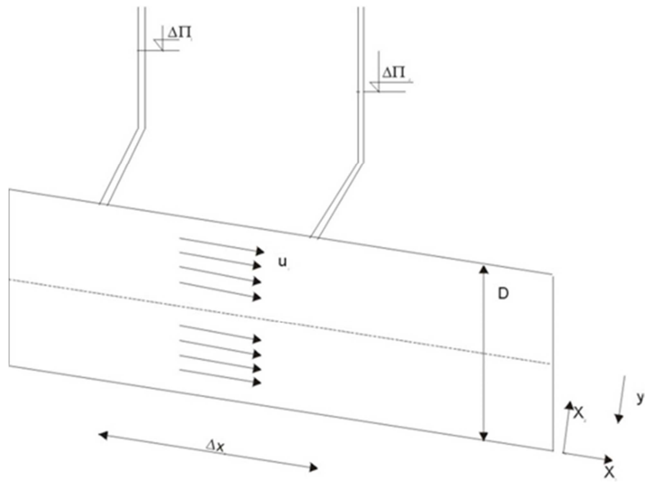

Figure 1. *Flow conductor.*

2. Existing Formulas

2.1. Uniform Laminar Axis Symmetrical Flow

Uncompressible fluid is analyzed in axis symmetrical conductor having as condition that flow is uniform, laminar and rectilinear, with diameter D and velocity u.

Navie – Stoks equation for round conductor:

$$0 = - \partial (gZ + p/\varrho)/\partial x_1 + \mu/\varrho\ (\partial^2 u_1/\partial x_2^2 + \partial^2 u_1/\partial x_3^2) \quad (2)$$

$$\partial\prod/\partial\ x_1 = \mu/\gamma\ (\partial^2 u/\partial x_2^2 + \partial^2 u/\partial x_3^2) \quad (3)$$

u – velocity
p – pressure
γ – specific weight
g – gravitational acceleration
ϱ - density
μ – viscous coefficient
∏ - piezometric head

Uncompressible fluid is analyzed:

$\partial u_i/\partial x_i=0$, $u_2=0$, $u_3=0$, $\partial u_1/\partial x_1=0$, $u_1=f(x_2, x_3)$

$\prod=f(x_1)$, so left and right side of equation have to be constant value.

Partial solution for velocity division in plane X_1OX_2, while $X_3=0$:

$$\partial\prod/\partial\ x_1 = \mu/\gamma\ \partial^2 u/\partial x_2^2 = - I_\Pi \quad (4)$$

$$\mu/\gamma\ \partial u/\partial x_2 = - I_\Pi\ x_2 + c_1 \quad (5)$$

$$\mu/\gamma\ u = - I_\Pi\ x_2^2/2 + c_1 x_2 + c_2 \quad (6)$$

Integrals constants are got from border conditions: $x_2=0$, u=0 and $x_2=2r$ u=0, (r – radius):

$c_1= I_\Pi\ r$, $c_2 = 0$

$$u= I_\Pi\ r^2\ \gamma/\mu\ (x_2/r - x_2^2/2r^2) \quad (7)$$

In the middle of flow $x_2=r$ $u=u_m$ - maximum velocity:

$$u_m = 1/2\ I_\Pi\ r^2\ \gamma/\mu \quad (8)$$

$$u/u_m = 2\ x_2/r - x_2^2/r^2 \quad (9)$$

$$u = u_m\ [1 - (2y/D)^2\ ] \quad (10)$$

where as x_2 is marked length from pipes wall and as y is marked length from pipes axis.

Analytic solution for velocity division is found, square parable as shown. [2]

2.2. Turbulence Axis Symmetrical Flow with Log Velocity Division

It is assumed log velocity division in turbulence layer validated with several experiments:

$$\overline{u} = \overline{u}_1 = c_I\ \sqrt{(\tau/\varrho)}\ \ln\ x_2/x_0 + \overline{u}_0 \quad (11)$$

c_I - constant experimentally determinate

$\sqrt{(\tau/\varrho)}$ - friction velocity describing influence of fluctuations on average velocity division

$\overline{u} = \overline{u}_0$ for $x_2 = x_0$ – border condition meaning velocity is known at length x_0 from pipes wall.

For value $x_2 = 0$ function is interruptible $u = - \infty$ therefor formula is not defined near conductors walls, defined in turbulence layer only.

For following analyze from several formulas is adopted formula for log relation in turbulence layer called "velocity deficit":

$$\overline{u}_m - \overline{u} = 2.5\ \sqrt{(\tau/\varrho)}\ \ln\ r/x_2 \quad (12)$$

Formula is solved for border condition in the middle of conductor $x_2 = D/2 = r$, $\overline{u}_0 = \overline{u}_m$ (maximum velocity).

$$\overline{u} = 2.5\ \sqrt{(\tau/\varrho)}\ \ln\ x_2/r + u_m \quad (13)$$

Formula derived for plane flow is validated also for axis symmetrical flow, is not defined in whole area of research.

For calculation adopted values are $\overline{u}_m$=1 m/s, D=0.1 m, $\overline{u} = 0.8\ \overline{u}_m = 0.8$ m/s, $\varrho = 1000$ kg/m^3

$\tau = ½\ \varrho\ \overline{u}^2 = 320$ Pa

$\sqrt{(\tau/\varrho)} = 0.566$

$$u = 2.5*\ 0.566\ \ln\ x_2/r + 1.0 \quad (14)$$

Calculated values are presented in *Table 1 and Figure 2*.

Table 1. *Log velocity calculation in cross – section.*

x_2 (m)	$y = x_2 - r$ (m)	$\overline{u}$ (m/s)
0	0.05	-
0.001	0.049	-
0.01	0.04	-
0.02	0.03	-
0.025	0.025	0.019
0.03	0.02	0.277
0.035	0.015	0.495
0.04	0.01	0.684
0.045	0.005	0.851
0.05	0.0	1.0

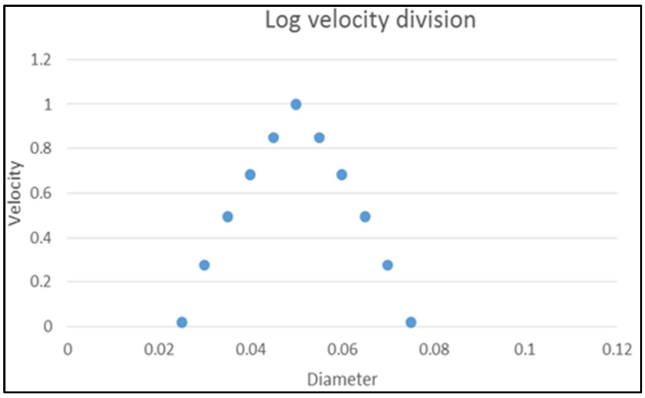

Figure 2. *Log velocity division – existing formula.*

Calculation was provided from wall to axis of conductor, from axis is counted symmetrical. [2]

2.3. Turbulence Axis Symmetrical Flow with Exponential Velocity Division

Based on experimental practice following exponential relation for velocity division is assumed:

$$\overline{u}/\ u_m = (x_2/r)^n \tag{15}$$

Coefficient n derived experimentally is followed velocity division.

From several exponential formulas is applied formula valid for smooth wall, meaning that laminar under layer existed on pipes wall where turbulence is not developed and having as known border condition $x_2 = \delta_c$ (fatness viscous under layer) and $u = u_c$ (velocity on under layers border). [2]

Applied formula is:

$$Y = 8.3\ X^{\ 1/7} \tag{16}$$

$Y = u_1/\sqrt{(\tau/\varrho)}$ and $X = x_2\sqrt{(\tau/\varrho)}/Y$

After variable are shifted formula is

$$u = 8.3^{7/8}\ (\sqrt{(\tau/\varrho)})^{9/8}\ x_2^{\ 1/8} \tag{17}$$

Adopted values are $\overline{u}_m$=1 m/s, D=0.1 m, $\overline{u} = 0.8\ \overline{u}_m = 0.8$ m/s, $\varrho = 1000$ kg/m^3, $c_\tau = 0.24$

$\tau = ½\ \varrho\ c_\tau\ \overline{u}^2 = 76.8$ Pa

$\sqrt{(\tau/\varrho)} = 0.277$

$$u = 1.503\ x_2^{\ 1/8} \tag{18}$$

Calculated values are presented in *Table 2* and *Figure 3*.

Table 2. *Exp velocity calculation in cross – section.*

x_2 (m)	$y = x_2 - r$ (m)	$\overline{u}$ (m/s)
0	0.05	0
0.001	0.049	0.63
0.01	0.04	0.84
0.02	0.03	0.92
0.03	0.02	0.97
0.04	0.01	1.00
0.05	0.0	1.03

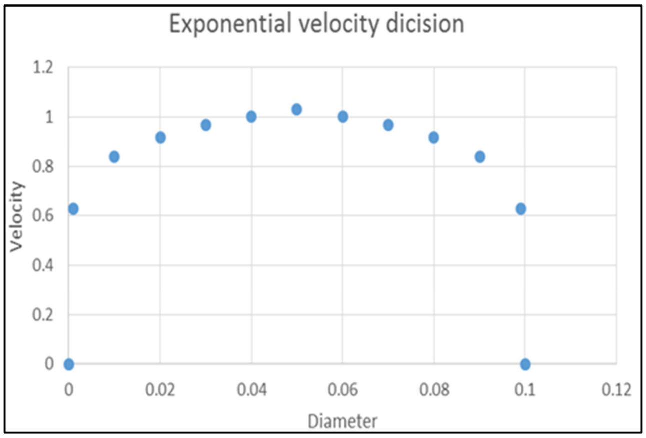

Figure 3. *Exp velocity division – existing formula.*

3. New Original Formulas

3.1. Turbulence Axis Symmetrical Flow with Log Velocity Division

Original log formula is presented for velocity division in axis symmetrical flow. Value $y = |\ x_2 - r\ |$ is involved in to calculation as length from axis conductor to walls, $0 < y \le r$, from axis is counted symmetrical (as used in laminar axis symmetrical flow). Function is interruptible at point $y = 0$, $u = +\infty$, therefor is adopted $u = u_m$ with sufficient currency. [8, 9]

$$u = -\ u_m\ \ln\ (2y/D)/12 \quad 0 < y \le r \quad y = |\ x_2 - r\ | \tag{19}$$

$y = 0$, $u = u_m$

Original formula (19) – A. RAJCEVIC

For calculation adopted values are $\overline{u}_m$=1 m/s, D=0.1 m, $\overline{u} = 0.8\ \overline{u}_m = 0.8$ m/s.

Calculated values are shown in *Table 3* and *Figure 4*.

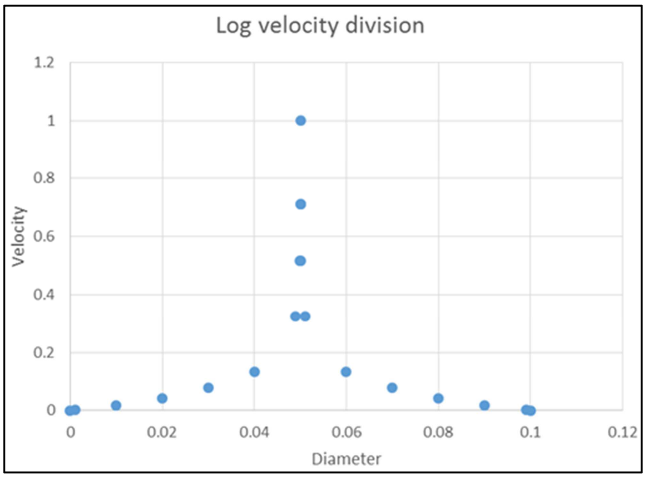

Figure 4. *Log velocity division – original formula.*

Table 3. *Log velocity calculation in cross – section.*

x_2 (m)	$y = x_2 - r$ (m)	$\overline{u}$ (m/s)
0	0.05	0
0.001	0.049	0.0017
0.01	0.04	0.0186
0.02	0.03	0.0426

x_2 (m)	$y = x_2 - r$ (m)	$\overline{u}$ (m/s)
0.03	0.02	0.0764
0.04	0.01	0.1341
0.049	0.001	0.326
0.0499	0.0001	0.5179
0.04999	0.00001	0.71
0.05	0.0	1.0

3.2. Turbulence Axis Symmetrical Flow with Exponential Velocity Division

Original exponential formula is presented for velocity division in axis symmetrical turbulence flow. Value $y = | x_2 - r |$ is involved in to calculation as distance from axis of conductor to walls, function is axis symmetrical.

$$u = 1.582\, u_m \left(1 - e^{2y/D}/e\right) \quad 0 < y \le r \quad y = | x_2 - r | \qquad (20)$$

Original formula (20) – A. RAJCEVIC

Adopted values are $\overline{u}_m$=1 m/s, D=0.1 m, $\overline{u} = 0.8\,\overline{u}_m = 0.8$ m/s.

Calculated values are shown in *Table 4* and *Figure 5*.

Table 4. *Exp velocity calculation in cross – section.*

x_2 (m)	$y = x_2 - r$ (m)	$\overline{u}$ (m/s)
0	0.05	0
0.001	0.049	0.0267
0.01	0.04	0.2828
0.02	0.03	0.5183
0.03	0.02	0.7137
0.04	0.01	0.8711
0.05	0.0	1.0

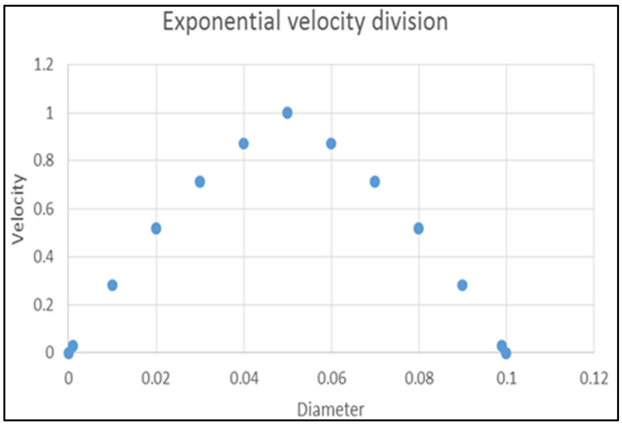

Figure 5. *Exp velocity division – original formula.*

4. Conclusion

Existing formulas for velocity division based on log and exponential functions have been analyzed and finally have concluded they have lot of deficiency in practice. Log formula for velocity division is not defined on whole interval of research. Exponential formula is suitable for flow with small velocity changes in the middle of pipe. [10, 11]

New presented formulas have significant advantages and removed in great deal mentioned lacks. Log formula for velocity division is enabled for calculation on whole interval of study, suitable for research with large velocity changes in conductors cross – section. Original formulas, log and exponential have to be validated experimentally and justified for use in some areas of fluid mechanics (compressible fluids, specific flows with abrupt velocity changes). [5, 6, 7, 12-15].

It could be concerned new original formulas are valid for plane flow between two plane boards.

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