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# Game Theoretic Approach to the Transmission and Prevalence of Alcohol Drinking Habits

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**Abstract:** Heavy alcohol consumption is known to be a major risk factor for disease and death globally. Various studies have related transmission of alcohol drinking habits with social contacts and peer pressure. Researchers have attempted to model this contact process through dynamical systems. Departing from this approach, this paper adopts a game theory model to study the transmission and prevalence of alcohol drinking habits, based on the concept of an evolutionarily stable population state. The proposed game theoretic model takes into account two scenarios. One deals with populations aged 15+ having two types of individuals: nondrinkers (N) and drinkers (D) whereas the other divides the same populations into nondrinkers (N), moderate drinkers (M) and heavy drinkers (H). In the former case, three types of pairwise interactions are possible between these individuals whereas in the latter, six types. The different possibilities inherent in these types are explained and the payoff matrices representing the interactions and the resulting gain expressions are presented. The game theoretic models are then analyzed and evolutionarily stable population states are computed for a few sets of parameter values. The advantage of this model is that it is found to be beneficial to understand the large time proportions of nondrinkers and drinkers in the population in a simpler manner than the dynamic system models. The work may be further expanded by dividing the existing classes of non-drinking and drinking populations on the basis of other critical aspects also.

**Keywords:** Game Theoretic Models, Evolutionary Games, Applications of Game Theory, Social Evolution

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## 1. Introduction

Mathematical models have been widely used to study the evolution and spread of various infectious diseases. These models are built mainly based on the contact process and other underlying phenomena that are inherent in an infectious disease. With the advancement of mathematical modeling, researchers have also attempted to model even habits and phenomena whose acquisition does not usually follow the patterns of typical epidemiologic contact process, but which result from some kind of human 'contact' process, which can even be 'conversion' contact social process. Models have been constructed for spread of fanatic behavior [7], smoking habits [33], transmission of rumors [20], crowd violence [28], the eating disorder disease 'bulimia nervosa' [13], drug use [38], alcohol drinking habits [4], 'music fever' [34], etc.

Alcohol is widely known as a psychoactive substance with dependence-producing properties whose harmful use ranks globally among the top five risk factors for disease, disability and death. The global alcohol per capita annual consumption among individuals aged 15+ is 6.2 liters of pure alcohol, whereas the same among drinking population is 17.2 liters. Worldwide 38.3% of the total population aged 15+ are current drinkers out of whom 16% are Heavy Episodic Drinkers (HED)<sup>1</sup> [39]. The influence of the family and friends is central, especially for adolescents and young adults, when analyzing the onset of alcohol use of an individual [8, 14, 27]. Various studies [1, 2, 11, 29, 30, 37] have found that a substantial part of the variance in regular drinking habits of individuals was explained by the drinking habits of family members and friends and the influence in most cases came from the closest friend [3, 5, 35, 36]. The very fact that

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<sup>1</sup> Heavy Episodic Drinking is defined as 60 or more grams of pure alcohol on at least one single occasion at least monthly

alcohol use is often initiated at a young age [12, 17, 18] and early onset of alcohol use increases the risk of alcohol abuse and life time dependence [15, 16] underlines the critical significance of the above findings. For most of the drinkers it is observed that inter-personal contacts and peer pressure play a major role in the onset of alcohol use. These findings lead one to the conclusion that alcohol drinking habits are ‘transmitted’ through social contacts. Dynamical models [4, 21, 22, 25, 26, 32] have been proposed previously for studying the effect of social contacts on transmission of alcohol drinking habits. In this paper a game theoretic approach to explain the transmission and prevalence of alcohol drinking habits in the population is introduced. This approach uses the concept of an evolutionarily stable population state introduced in [23, 24] and is motivated by earlier successful applications of game theory in various biological settings [6]. The objective in this paper is to initiate the use of game models in explaining transmission and prevalence of ‘social contact habits’ like alcohol consumption.

## 2. Description of the Game Theoretic Model

The proposed game theoretic model takes into account two scenarios. The first one (2.1) deals with populations aged 15+ having two types of individuals: nondrinkers (N) and drinkers (D) whereas the other scenario (2.2) divides populations aged 15+ into nondrinkers (N), moderate drinkers (M)<sup>2</sup> and heavy drinkers (H).

### 2.1. The $2 \times 2$ Model

A population aged 15+ consisting of two types of individuals is considered: nondrinkers (N) and drinkers (D). Three types of pairwise interactions are possible between these individuals, namely those of nondrinkers with nondrinkers (NN), nondrinkers with drinkers (ND) and drinkers with drinkers (DD). In each such interaction the gain of an individual is assumed to be a combination of complete cessation of drinking habits, the advantages (health, monetary, etc.) of being a nondrinker, or the subjective pleasure of being a drinker. On the other hand, the loss of an individual is perceived to be the disadvantages of being a drinker. When a nondrinker interacts with a nondrinker (NN interaction) no gain/loss is expected through reduction/acquisition of drinking levels; but there is a reinforced health and monetary advantage to these individuals which is quantified as  $b_n$ . In DD interactions, though the cessation of drinking habit is unlikely, both drinkers have other disadvantages such as health and monetary decline, which will be quantified as  $\ell_d$ . If a drinker assigns more weight to the pleasure aspects of drinking than health/monetary aspects, then  $\ell_d$  will be taken to be negative.

As a result of ND interaction, there are four possibilities respectively occurring with probabilities  $p_i$ ,  $i = 1, 2, 3, 4$ . In each case the average gain of nondrinkers and drinkers are

denoted by  $g_{nd}^i(n)$  and  $g_{nd}^i(d)$ ,  $i = 1, 2, 3, 4$  respectively. In the first case, nondrinker will have no change while the drinker would start behaving as nondrinker ( $ND \rightarrow NN$ ). The average gain for nondrinker in this case is  $g_{nd}^1(n) = \frac{1}{2}(b_n + c_{dn})$ , which is the average of the benefit  $b_n$  of being a nondrinker and the gain  $c_{dn}$  of a drinker behaving as a nondrinker. Clearly as no drinker remains after this type of interaction, the gain of a drinker  $g_{nd}^1(d)$  is taken to be 0. Similarly in the second case where drinker will have no change while nondrinker becomes a drinker ( $ND \rightarrow DD$ ),  $g_{nd}^2(n) = 0$ , and  $g_{nd}^2(d) = \frac{1}{2}(-\ell_d - d_{nd})$ , where  $d_{nd}$  denotes the loss of a nondrinker when becoming a drinker. The third case corresponds to the situation where both nondrinker and drinker getting changed ( $ND \rightarrow DN$ ). Here  $g_{nd}^3(n) = c_{dn}$  and  $g_{nd}^3(d) = -d_{nd}$ . In the last case no change occurs for both nondrinker and drinker ( $ND \rightarrow ND$ ). Here  $g_{nd}^4(n) = b_n$  and  $g_{nd}^4(d) = -\ell_d$ . Therefore the average gains of nondrinkers and drinkers resulting from ND interactions are  $\sum_{i=1}^4 p_i g_{nd}^i(n)$  and

$\sum_{i=1}^4 p_i g_{nd}^i(d)$  respectively.

The above mentioned types of interactions and the resulting gains can be represented using the following payoff matrix:

$$A = \begin{matrix} & \begin{matrix} N & D \end{matrix} \\ \begin{matrix} N \\ D \end{matrix} & \begin{pmatrix} b_n & \sum_{i=1}^4 p_i g_{nd}^i(n) \\ \sum_{i=1}^4 p_i g_{nd}^i(d) & -\ell_d \end{pmatrix} \end{matrix}$$

### 2.2. The $3 \times 3$ Model

In the scenario of this subsection, the population aged 15+ consists of three types of individuals: nondrinkers (N), moderate drinkers (M) and heavy drinkers (H). As a result, six types of pairwise interactions are possible. In each such interaction the gain of an individual is assumed to be a combination of reduction in his/her drinking levels, complete cessation of drinking habits, the advantages (health, monetary, etc.) of being a nondrinker, or the subjective pleasure of being a drinker. As in the  $2 \times 2$  model (2.1), the loss of an individual is the increase in drinking levels or the disadvantages of being a drinker. In the NN interaction no gain/loss is expected through reduction/acquisition of drinking levels; however there is a reinforced health and monetary advantage  $b_n$ . In MM(HH) interactions, though the reduction/increase in drinking levels are unlikely, both drinkers have other disadvantages such as health and monetary decline, which are denoted by  $\ell_m(\ell_h)$ . As in the previous subsection, if a drinker assigns more weight to the pleasure aspects of drinking than health/monetary aspects, then  $\ell_m$  and  $\ell_h$  will be taken to be negative.

There are four possibilities in NM interaction, respectively occurring with probabilities  $p_i$ ,  $i = 1, 2, 3, 4$ . In each case the

<sup>2</sup> For women(men), moderate drinking is usually defined as no more than 3(4) drinks on any single day and no more than 7(14) drinks per week.

average gain of nondrinkers and moderate drinkers is denoted by  $g_{nm}^i(n)$  and  $g_{nm}^i(m)$ ,  $i = 1, \dots, 4$  respectively. In the first possibility, nondrinker will have no change while the moderate drinker would behave as nondrinker ( $NM \rightarrow NN$ ). The average gain for nondrinker in this case is  $g_{nm}^1(n) = \frac{1}{2}(b_n + c_{mn})$ , which is the average of the benefit  $b_n$  of being a nondrinker and the gain  $c_{mn}$  of a moderate drinker behaving as a nondrinker. As in the  $2 \times 2$  model, as no moderate drinker remains after this type of interaction, the gain of moderate drinker  $g_{nm}^1(m)$  is taken to be 0. Similarly in the second case where moderate drinker will have no change while nondrinker assumes the drinking role of moderate drinkers ( $NM \rightarrow MM$ ),  $g_{nm}^2(n) = 0$ , and  $g_{nm}^2(m) = \frac{1}{2}(-\ell_m - d_{nm})$ , where  $d_{nm}$  denotes the loss of a nondrinker when becoming a moderate drinker. The third possibility corresponds to the situation where both nondrinker and moderate drinker getting changed ( $NM \rightarrow MN$ ). Here  $g_{nm}^3(n) = c_{mn}$  and  $g_{nm}^3(m) = -d_{nm}$ . In the last case both nondrinker and moderate drinker remain unchanged ( $NM \rightarrow NM$ ). Here  $g_{nm}^4(n) = b_n$  and  $g_{nm}^4(m) = -\ell_m$ . This leads the average gain expressions of nondrinkers and moderate drinkers as  $\sum_{i=1}^4 p_i g_{nm}^i(n)$  and  $\sum_{i=1}^4 p_i g_{nm}^i(m)$  respectively.

Similarly as in the previous paragraph, in NH interaction also, there are four possibilities respectively occurring with probabilities  $q_i, i = 1, 2, 3, 4$ . In each case the average gain of nondrinker and heavy drinker are denoted by  $g_{nh}^i(n)$  and  $g_{nh}^i(h)$ ,  $i = 1, 2, 3, 4$ . respectively. In the first type of transition, nondrinker will have no change while the heavy drinker behaves as a nondrinker ( $NH \rightarrow NN$ ). The average gain for nondrinker in this case is  $g_{nh}^1(n) = \frac{1}{2}(b_n + c_{hn})$ , which is the average of the benefit  $b_n$  of being a nondrinker and the gain  $c_{hn}$  of a heavy drinker behaving as a nondrinker and  $g_{nh}^1(h)$  is taken to be 0. In the second case where heavy drinker will have no change while nondrinker assumes the drinking behavior of heavy drinker ( $NH \rightarrow HH$ ),  $g_{nh}^2(n) = 0$ , and  $g_{nh}^2(h) = \frac{1}{2}(-\ell_h - d_{nh})$ , where  $d_{nh}$  denotes the loss of a nondrinker when behaving as a heavy drinker. Corresponding to the case where both nondrinker and heavy drinker getting changed ( $NH \rightarrow HN$ ),  $g_{nh}^3(n) = c_{hn}$  and  $g_{nh}^3(h) = -d_{nh}$ . When no change occurs for both nondrinker and heavy drinker, ( $NH \rightarrow NH$ ),  $g_{nh}^4(n) = b_n$  and  $g_{nh}^4(h) = -\ell_h$ . In this possibility, the average gain of nondrinkers and heavy drinkers is  $\sum_{i=1}^4 q_i g_{nh}^i(n)$  and  $\sum_{i=1}^4 q_i g_{nh}^i(h)$  respectively.

In the final type of interactions (MH), four possibilities respectively occurring with probabilities  $r_i, i = 1, 2, 3, 4$  are considered. In each case the average gain of moderate drinkers and heavy drinkers is denoted by  $g_{mh}^i(m)$  and  $g_{mh}^i(h)$ ,  $i = 1, 2, 3, 4$ , respectively. As in the above types of interactions, the gain expressions as obtained as follows:

$$g_{mh}^1(m) = \frac{1}{2}(-\ell_m + c_{hm}),$$

$$g_{mh}^1(h) = 0, \quad g_{mh}^2(m) = 0,$$

$$g_{mh}^2(h) = \frac{1}{2}(-\ell_h - d_{mh}),$$

$$g_{mh}^3(m) = c_{hm}, \quad g_{mh}^3(h) = -d_{mh},$$

$$g_{mh}^4(m) = -\ell_m,$$

$$g_{mh}^4(h) = -\ell_h,$$

$$\sum_{i=1}^4 r_i g_{mh}^i(m)$$

and

$$\sum_{i=1}^4 r_i g_{mh}^i(h).$$

These interactions and the resulting gain expressions can be represented using the following payoff matrix:

$$A = \begin{matrix} & \begin{matrix} N & M & H \end{matrix} \\ \begin{matrix} N \\ M \\ H \end{matrix} & \begin{pmatrix} b_n & \sum_{i=1}^4 p_i g_{nm}^i(n) & \sum_{i=1}^4 q_i g_{nh}^i(n) \\ \sum_{i=1}^4 p_i g_{nm}^i(m) & -\ell_m & \sum_{i=1}^4 r_i g_{mh}^i(m) \\ \sum_{i=1}^4 q_i g_{nh}^i(h) & \sum_{i=1}^4 r_i g_{mh}^i(h) & -\ell_h \end{pmatrix} \end{matrix}$$

In the next section an evolutionarily stable state (ESS) of a matrix game is defined and its relevance to the game models depicted above is explained.

### 3. ESS and Its Computation

#### 3.1. Description of ESS

A population state  $x = (x_1, x_2, \dots, x_k)$  in a  $k$  by  $k$  matrix game represents the proportions of  $k$  types of individuals in the whole population. One is interested in finding population states which are “evolutionarily stable” in the following sense:

**Definition 3.1.** [23, 24] A population state  $x^* = (x_1^*, x_2^*, \dots, x_k^*)$  in the matrix game  $A$  is said to be an evolutionarily stable state (ESS) if for every mutant state  $x \neq x^*$ , there is  $\epsilon^* \in (0, 1)$  such that

$$x^* A (\epsilon x + (1 - \epsilon) x^*)^T > x A (\epsilon x + (1 - \epsilon) x^*)^T, \quad 0 < \epsilon < \epsilon^*.$$

It may be noted that a population state  $x^*$  is an ESS exactly when it can withstand mutations arising in small proportions. The concept of ESS may also be considered as a refinement of Nash equilibria [10, 31]. The next theorem gives a set of necessary and sufficient conditions for a population state to be an ESS.

**Theorem 3.1.** [9, 19] A population state  $x^*$  is an ESS if and only if the following two conditions are satisfied:

- (E1)  $(Ax^{*T})_i \leq x^*Ax^{*T}$ ;  $i = 1, 2, \dots, k$ ,  
 (E2)  $x^*Ay^T > yAy^T$  whenever  $y \neq x^*$  and  $x^*Ax^{*T} = yAx^{*T}$ .

The next theorem [9, 19] is more useful in analyzing the  $2 \times 2$  model:

**Theorem 3.2.** Let  $k=2$  and  $A = [a_{ij}]$  be the payoff matrix. Assume that  $a_{11}a_{22} \neq 0$ . Then exactly one of the following holds true:

- (F1) If  $a_{11} > a_{21}$  and  $a_{22} > a_{12}$ , then the game has exactly two ESS, namely  $(1,0)$ ,  $(0,1)$ .  
 (F2) If  $a_{11} < a_{21}$  and  $a_{22} < a_{12}$ , then the game has exactly a unique ESS given by

$$\left( \frac{a_{22} - a_{12}}{a_{11} - a_{21} + a_{22} - a_{12}}, \frac{a_{11} - a_{21}}{a_{11} - a_{21} + a_{22} - a_{12}} \right).$$

- (F3) If  $a_{11} > a_{21}$  and  $a_{22} < a_{12}$ , then  $(1,0)$  is the only ESS.  
 (F4) If  $a_{11} < a_{21}$  and  $a_{22} > a_{12}$ , then  $(0,1)$  is the only ESS.

Using Theorems 3.1 and 3.2 the above game theoretic models are analyzed and evolutionarily stable population states

for a few sets of parameter values are found:

### 3.2. Computation of ESS

To apply Theorem 3.2 in the context of  $2 \times 2$  model, it may be noted that

$$\begin{aligned} a_{11} &= b_n, \\ a_{12} &= b_n\left(\frac{p_1}{2} + p_4\right) + c_{dn}\left(\frac{p_1}{2} + p_3\right), \\ a_{21} &= -\ell_d\left(\frac{p_2}{2} + p_4\right) - d_{nd}\left(\frac{p_2}{2} + p_3\right), \\ a_{22} &= -\ell_d. \end{aligned} \quad (1)$$

**Example 3.1.**

In this example, the  $2 \times 2$  model where the benefits of being a nondrinker ( $b_n$ ), the loss of being a drinker ( $\ell_d$ ), the gain of a drinker becoming nondrinker ( $c_{dn}$ ) and the loss of a nondrinker becoming a drinker ( $d_{nd}$ ) are all positive, is considered. This indicates that the health/monetary decline due to drinking is given more importance than the pleasure benefits due to drinking. From (1) clearly  $a_{11} > a_{21}$  and  $a_{22} < a_{12}$ , which, by theorem 3.2 imply that the only ESS is  $x^* = (1,0)$ . This is consistent with the reality that all individuals in the population tend to be nondrinkers when drinking is realized as harmful.

**Example 3.2.**

In the  $2 \times 2$  model where

$$b_n < -\left(\ell_d\left(\frac{p_2}{2} + p_4\right) + d_{nd}\left(\frac{p_2}{2} + p_3\right)\right)$$

and

$$-\ell_d < \left(b_n\left(\frac{p_1}{2} + p_4\right) + c_{dn}\left(\frac{p_1}{2} + p_3\right)\right),$$

From theorem 3.2 one has the unique ESS,

$$x^* = (x_1^*, 1 - x_1^*),$$

$$x_1^* = \frac{-\left(\ell_d + b_n\left(\frac{p_1}{2} + p_4\right) + c_{dn}\left(\frac{p_1}{2} + p_3\right)\right)}{\left(b_n + \ell_d\left(\frac{p_2}{2} + p_4\right) + d_{nd}\left(\frac{p_2}{2} + p_3\right)\right) - \left(\ell_d + b_n\left(\frac{p_1}{2} + p_4\right) + c_{dn}\left(\frac{p_1}{2} + p_3\right)\right)}.$$

For instance, one obtains  $x^* = (0.6054, 0.3946)$  when  $b_n = 1$ ,  $\ell_d = -2$ ,  $c_{dn} = 20$ ,  $d_{nd} = -5.5$ ,  $p_1 = 0.25$ ,  $p_2 = 0.35$ ,  $p_3 = 0.05$ ,  $p_4 = 0.35$ . This means that the stable population state  $x^*$  consists of 61% nondrinkers and 39% drinkers approximately, which is very close to the global proportions of nondrinkers and drinkers [39].

In the next three examples the  $3 \times 3$  model using Theorem 3.1 is illustrated.

**Example 3.3.**

It is assumed  $b_n = 1$ . The loss of being a moderate drinker is assumed to be higher in magnitude than the gain of being a nondrinker, and hence  $\ell_m = 2$  is taken. By a similar reasoning  $\ell_h = 4$  is taken. The probabilities of various cases occurring in NM interactions are taken to be  $p_1 = 0.2$ ,  $p_2 = 0.3$ ,  $p_3 =$

0.1, and  $p_4 = 0.4$ . In this example, the probabilities  $q_i$ ,  $r_i$  are also taken to be the same as  $p_i$ . As the assumption is that the benefit of a moderate drinker becoming a nondrinker does not exceed the same for an already nondrinker,  $c_{mn} = 0.7$  is taken. Likewise,  $c_{hn} = 0.6$  and  $c_{hm} = 0.4$  are also taken. Analogous arguments lead us to the choices of  $d_{nm} = 1.5$ ,  $d_{nh} = 3$ , and  $d_{mh} = 2.5$ . Thus our pay off matrix becomes

$$A = \begin{bmatrix} 1 & 0.64 & 0.50 \\ -1.475 & -2 & -0.92 \\ -2.95 & -2.825 & -4 \end{bmatrix}.$$

It can be verified that the only ESS for this set of parameter values is  $x^* = (1, 0, 0)$ . This means that in a population of

nondrinkers, moderate and heavy drinkers appearing in small proportions gain less than that of nondrinkers.

*Example 3.4.*

In this example also, take  $b_n = 1$ . The scenario wherein individuals give more weight to pleasure arising out of drinking than its health/monetary disadvantages is considered, and hence  $\ell_m$  and  $\ell_h$  are taken to be -3 and -4 respectively. The probabilities of various cases occurring in NM, NH and MH interactions are assumed to be  $p_1 = 0.4$ ,  $p_2 = 0.3$ ,  $p_3 = 0.05$ ,  $p_4 = 0.25$ ,  $q_1 = 0.3$ ,  $q_2 = 0.2$ ,  $q_3 = 0.1$ ,  $q_4 = 0.4$ ,  $r_1 = 0.3$ ,  $r_2 = 0.2$ ,  $r_3 = 0.15$ , and  $r_4 = 0.35$ .

The remaining parameters are  $c_{mn} = 20$ ,  $c_{hn} = 28$ ,  $c_{hm} = 26$ ,  $d_{nm} = -9.5$ ,  $d_{nh} = -10$ , and  $d_{mh} = -12$ .

Note that relatively higher values for  $c_{mn}$ ,  $c_{hn}$ ,  $c_{hm}$  indicate that individuals are influenced by various means to curtail their drinking habits even while they seek pleasure in the same. The negative values of  $d_{nm}$ ,  $d_{nh}$  and  $d_{mh}$  point to the prevalent assumption of drinkers about the inherent pleasure attached to drinking. These parameters yield the payoff matrix as

$$A = \begin{bmatrix} 1 & 5.45 & 7.55 \\ 3.1 & 3 & 9.3 \\ 4 & 4.8 & 4 \end{bmatrix}.$$

Using Theorem 3.1 it can be shown that the only ESS is  $x^* = (0.3448, 0.4454, 0.2098)$ . This stable population state consists of approximately 34% nondrinkers, 45% moderate drinkers and 21% heavy drinkers.

*Remark 3.1.* Even though the above two  $3 \times 3$  examples may not reflect the actual proportions of nondrinkers and drinkers, they serve the purpose of illustrating the game theoretic model explained in the previous section. The next example captures the social drinking scenario in a more realistic way.

*Example 3.5.*

This example is obtained by slightly changing some of the parameter values given in the previous example. More precisely,

$b_n = 1$ ,  $\ell_m = -2.8$ ,  $\ell_h = -3.9$ ,  $p_1 = 0.5$ ,  $p_2 = 0.2$ ,  $p_3 = 0.05$ ,  $p_4 = 0.25$ ,  $q_1 = 0.45$ ,  $q_2 = 0.25$ ,  $q_3 = 0.1$ ,  $q_4 = 0.2$ ,  $r_1 = 0.55$ ,  $r_2 = 0.1$ ,  $r_3 = 0.05$ ,  $r_4 = 0.3$ ,  $c_{mn} = 20$ ,  $c_{hn} = 28$ ,  $c_{hm} = 27$ ,  $d_{nm} = -9.5$ ,  $d_{nh} = -10$ , and  $d_{mh} = -14.5$ .

The payoff matrix is

$$A = \begin{bmatrix} 1 & 6.5 & 9.525 \\ 2.4050 & 2.8 & 10.385 \\ 3.5175 & 2.815 & 3.9 \end{bmatrix}$$

and the unique ESS is  $x^* = (0.6280, 0.2637, 0.1083)$ . Here it is observed that the stable population state contains approximately 63% nondrinkers, 26% moderate drinkers and 11% heavy drinkers which closely resembles the actual proportions mentioned in the introduction [39]. In the next section the replicator dynamics associated with the game is dealt with and ESS as limit of its trajectories is obtained.

## 4. ESS as Limit of Replicator Trajectories

The replicator dynamics corresponding to the matrix game  $A$  is given by

$$\dot{x}_i = x_i[(Ax^T)_i - xAx^T], \quad i = 1, 2, \dots, k. \quad (2)$$

The next result is helpful to interpret ESS as limit of replicator trajectories:

*Theorem 4.1.* [9, 19] For the replicator dynamics 2, one has the following:

1. The simplex of all population states and its faces are forward invariant.
2. If  $x^*$  is an ESS, then it is an asymptotically stable state of 2. If in addition  $x_i(0) > 0$  for  $i = 1, 2, \dots, k$ , then the basin of attraction of  $x^*$  is the interior of the simplex of populations states.

## 5. Concluding Remarks

It is a known phenomenon that alcohol drinking habits are to a great extent transmitted through social contacts and peer pressure. Motivated by this observation, different from the existing dynamic models based on differential/difference equations, in this paper our objective has been to introduce a game theoretic model to explain the prevalence and transmission of alcohol drinking habits. The concept of ESS is used to identify the stable population states in our game theoretic model. For two general classes of  $2 \times 2$  model, the proposed game approach has been illustrated. Furthermore, for three sets of specific parameter values, after computations using Theorem 3.1, stable population states for the  $3 \times 3$  model are obtained. In particular the stable population states  $x^* = (0.6054, 0.3946)$  and  $x^* = (0.6280, 0.2637, 0.1083)$  in examples 3.2 and 3.5 reflect the proportions, available in global reports [39], of nondrinking and drinking classes in the population very closely. Other examples point out how the proportions of nondrinking and drinking classes vary with the changes in the model parameters. Parameters used in these examples are hypothetical and their accurate estimation is essential for the more realistic predictions using this model. An obvious advantage of this game theoretic approach is that it is comparatively much simpler than its dynamic model counterparts in understanding the large time population state. An interesting future work is to introduce more explicit dynamic models into this game theoretic approach using various game dynamics. Furthermore, one can divide the existing classes of non-drinking and drinking populations on the basis of other critical aspects such as sex and age to explain the transmission and prevalence of drinking habits at a finer level.

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