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Application of Linear Programming for Decision Making to Business in Whao Beverages Nig. Ltd

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Abstract: Linear Programming is a scientific method used for obtaining and determine the most optimal solution for a problem with given constraints. This technique is useful for guidance of a quantitative decision-making in different fields such as business planning, industrial engineering, production firm and to the extent of social and physical sciences. This research examines the application of linear programming to determine the product(s) that maximize(s) the profit of WHAO Beverages Nig. Ltd. The data used for this research is a secondary data obtained from the company. The method used to analyze the data is a revised simplex method of a linear programming. The findings in this research was able to formulate mathematical model for the research problem; determine the product that will yield the maximum profit for the company; and know the quantity of this product that will give this profit. Data obtained from the company undergoes two iterations, whose analysis shows that the squash drinks maximize the profit of the company at the second iteration. The quantity of SQUASH Drinks that gave the maximum profit of 264 units produced and the maximum profit of the company (WHAO Beverages Nig. Ltd.) was #2200.83.

Keywords: Linear Programming, Simplex Method, Maximization, Resources, Business, Portfolio, Decision-Makers, Optimization

1. Introduction

Many managerial or decisional problems are concerned with how limited resources can be allocated efficiently to meet the desired objective i.e. if resources of any kind be it capital, labour, machines, materials are being allocated on this limited resources, then it is necessary to determine the most effective and profitable way of allocating the resources to meet people's large demands. After many decades of debate within the economic profession and beyond, the profit maximization hypothesis remains controversial. Few today will argue that profit maximization is a literal description of what all managers or decision makers in firms pursue.

In a world where some managers may maximize profit and others may not, debate is largely focused on arguments associated with natural selection. Going back as far as Wagner H. M [19] some economists have argued that firms in which decision making most closely resemble profit maximization i.e. what to produce, division of labour and price of the goods, decisions will be more likely to survive, and hence it can safely be assumed that industry outcomes are consistent with profit maximization, regardless of the actual motives of individual managers. Because intentions may no longer matter, natural selection arguments are often hard to rebut. The idea that natural selection favours profit maximization has been widely challenged, perhaps most notably by Pedro and Stefan S. [14].

Theoretically, Hamdy A. T [8] shows that profit maximization leads to certain bankrupting of the firm, while alternation strategies have a positive probability of long term survival. Many producers think of a better way to minimize their cost of production so as to maximize their profit margin. Any decision that a firm must take to achieve these goals are susceptible to statistical analysis and evaluation by linear programming.

An Entrepreneur is a person who performs entrepreneurial function. In line with Ogundele and Olayemi study [11], they define Entrepreneurs as an agent of social and economic change. It has been asserted by Akiniyi [1] that the growth and expansion of a business depends critically on the decision of the entrepreneurs. Those decisions include pricing decision, the identification of target market, ensuring standardization of the company's product, the level of technology involvement, manpower and others. Every entrepreneur is faced with the task of making decision as regards to efficient allocation of the available resources at his disposal to various areas of need of the concerned organization. This is necessary and sufficient so as to minimize the cost incurred while maximizing profit earned. Such resources allocation decisions are adopted through the use of quantitative technique such as Linear programming. Simplest Tableau, Transportation model, Net work analysis, sensitivity analysis and a host of others.

Linear programming according to Heizer and Render [9] is a mathematical technique used to rationalize many managerial decisions as regards the allocation of economic resources. Akiniyi [1] also defined it as an optimization instrument used to analyze constrained optimization problems in which the objective function is a linear function which can be maximized or minimized subject to Linear inequality constraints.

Optimization is the way of life. We all have finite resources and time and we want to make the most of them. From using your time productively to solving supply chain problems for your company – everything uses optimization. It's an especially interesting and relevant topic in data science. It is also a very interesting topic – it starts with simple problems, but it can get very complex. For example, sharing a bar of chocolate between siblings is a simple optimization problem. We don't think in mathematical terms while solving it. On the other hand, devising inventory and warehousing strategy for an e-tailer can be very complex. Millions of SKUs with different popularity in different regions to be delivered in defined time and resources – you see what I mean!

Linear programming (LP) is one of the simplest ways to perform optimization. It helps you solve some very complex optimization problems by making a few simplifying assumptions. As an analyst, you are bound to come across applications and problems to be solved by Linear Programming.

For some reasons, Linear Programming does not get as much attention as it deserves in learning data science. As such, we thought of doing justice to this awesome technique by writing an article that explains Linear programming in simple English. Hence we have kept the content as simple as possible. The idea is to get you started and excited about Linear Programming.

2. Objectives

The objectives of this research are to:

1) formulate a mathematical model for the research problem.
2) determine the variant drink (product) that will give (yield) the maximum profit for the company.
3) determine the maximum profit of the company from this product.
4) know the quantity of this product that will give this profit.

3. Literature Review

Meng and Yang [15] discussed various linear programming applications and techniques. The authors reported that aggregate production planning is the most important aspect of linear programming analysis. Markowitz [7] discussed the financial research and investment aspect of the portfolio optimization problem as one of the standard and most important aspects of portfolio management. An elegant way of managing risk in financial markets known as the portfolio theory was introduced in the study. Konno and Yamazaki [6] discussed Markowitz's model and assessed it as a single-period model where the investor's main objective was to maximize the portfolio's expected return. William introduced and discussed the Capital Asset Pricing Model (CAPM) based upon the empirical observation that gives the maximum expected return. The author compared and analyzed Markowitz's model to develop a simplified variant that reduces data and computational requirements. Such an empirical fact was analyzed, discussed, and supported by William.

Also, Ogryezak [18] discussed and analyzed the procedures and guidelines of setting investment policy. An asset allocation model was proposed and developed where the expected return for an asset class will be estimated using the simplex algorithm as an application of linear programming. Marcus [4] presented and analyzed price efficiency as a market where all available information that is relevant to the valuation of securities at all times is fully reflected by the price. Olayinka et al. [13] examined and applied linear programming techniques in the entrepreneur decision-making process to maximize profits with the available resources through a fast-food firm with the challenges of product selection and profit maximization due to an increase in the price of raw materials. Oladejo et al. [12] considered the importance of optimization principles in maximized profits and minimized cost of production and applied linear programming techniques in solving a particular challenge in a bakery production firm using the AMPL software.

Thus, in this paper, we examine the level of investment in a selected portfolio that gives maximum returns with minimal input based on the secondary data supplied by a particular firm, which were used as the parameters for the proposed linear programming model. This study has not been previously examined, and it has created gaps in portfolio management and optimization of a firm. The study is motivated by the earlier reports on the portfolio optimization and risk management of firms. A sensitivity analysis to ascertain the robustness of the resulting model towards the changes in input parameters to determine how redundant a constraint linear programming is carried out. The availability of funds and the allocation of each component of the portfolio to maximize returns and minimize inputs by portfolio holders and managers who are the major decision-makers in allocating their resources were also determined following Olayinka et al.'s [13] and Oladejo et al.'s [12] linear programming techniques.

There seem to be paucity of Literature on the application of Linear programming to the entrepreneur decision making process. However the few studies selected are reviewed below. The Linear programming components were also identified by Taha [17] and Stevenson [16]. The components according to them are the proportionality, additively, non-negativity, linearity, deterministic and with fixed external factors.

Finally, according to the Chartered Institute of Management Accounting, 2009, the technique designed to assist management in solving optimizations problem subject to limitation or imposed restriction. There are diverse opinions on application of simplex method to make decision in management in different sectors. During the world war he worked on planning methods for the US Air Force. Dantzig [5] formulated linear inequalities inspired by Wassily Leontief. After that he planned for solving the industrial and business problems. Initially, Dantzig didn't include objectives in formulation so that huge number of feasible solution found, therefore more rules were required to choose a best solution among all feasible solution, In Mid 1947 Dantzig included objectives in his formulation. Afterwards, he developed a "Simplex Method" to solve linear programming problems. Simplex method is a simple, elegant, yet powerful tool for solving linear programming problems. Simplex is used to solve the major problems in many different fields like optimize maximum profit, minimize cost, agriculture, human resources and manufacturing decision making etc. Limited data is required for calculate the result by using Simplex method which is easily available. Today's most powerful simplex solver for excel is used. In 1993, solver engineering was created and since 1995, solver has been supplying and dealing with implementation of R3 management system of SAP company. This famous method for LP is used in standard Excel solver and developer of solver built into optimization and premium solver platform. This technology can handle up to 8,000 variables and 8,000 constraints and it is much faster and gives automatically best presolve strategy. Thousands of the companies are using Simplex method and solver as in this review paper showed the different application in many areas. Most researchers such as Kurtz [10], Taha [17], Benedict. I. Ezema [3] posit that the use of scientific methods, particularly linear programming in the allocation of scarce resources is play vital role to the manufacturing to boost the output.

4. Methodology

A systematic procedure on the choice and rationale pertaining to all decisions in planning and implementation of strategies that will be adopted to collect data and undertake analytical aspects using Revised simplex method. Linear Programming**:** Linear programming deals with the optimization (maximization or minimization) of a function of variables known as objective functions. It is subject to a set of linear equalities and/or inequalities known as *constraints.* Linear programming is a mathematical technique which involves the allocation of limited resources in an optimal manner on the basis of a given criterion of optimality. The most widely used method for solving Linear Programming problems consisting of any number of variables is called the simplex method developed by G. Dantzig in 1947 and made generally available in 1951.

5. Data Analysis and Interpretation

The revised simplex method of linear programming was used for the analysis. However, the simplex method was also used to run to the data. This is to confirm the output of the different method of data analysis.

5.1. Data

Table 1. *Production Data.*

Products	Raw material consumed per week (kg)	Total available raw material	Production output per week (pcs)	Raw material consumed per unit/week (kg)	Required labour time per hrsweek (min)	Available labour time per week (hrs)	Total Available labour time per week (hrs)	Required machine time per unit / week (min)	Available machine time per week (hrs)	(scp) per unit (#)	(sp) per unit (#)	Profit
X_1	400	1000	7200	18.00	0.50	48	96	0.70	96	22.50	25	2.50
X_2	500	1000	10800	21.60	0.43	41	96	0.50	96	29.17	33.33	4.16
X_3	700	1000	14400	20.57	0.38	36	96	0.38	96	29.17	33.33	4.16
X_4	150	250	2880	19.20	1.13	108	96	1.25	96	58.33	66.67	8.34
X_5	200	250	4320	21.60	0.83	80	96	1.00	96	58.33	66.67	8.34
X_6	225	300	3240	14.40	1.11	107	96	1.11	96	58.33	66.67	8.34

Source: Whao Beverages Production.

5.2. Model Formulation

Table 2. *Productions' Raw materials, labour time, machine time and profit.*

Product	Raw Material	Labour Time	Machine Time	Profit
PineappleDrink (X_1)	18.00	0.55	0.70	2.50
Orange superb drink (X_2)	21.60	0.43	0.50	4.16
Superb AppleDrink (X_3)	20.57	0.38	0.38	4.16
Tropical drink (X_4)	19.20	1.13	1.25	8.34
Cream Soda (X_5)	21.60	0.83	1.00	8.34
Squash drink (X_6)	14.40	1.11	1.11	8.34
Total	3800	516	576	

Using the Revised Simplex Method, the model in standard form will be;

$$\text{Max } Z - 2.50\, X_1 - 4.16\, X_2 - 4.16\, X_3 - 8.34\, X_4 - 8.34\, X_4 - 8.34\, X_6$$

S.t

$$18.00\, X_1 + 21.60\, X_2 + 20.57\, X_3 + 19.20\, X_4 + 21.60\, X_5 + 14.40\, X_6 = 3800$$

$$0.55\, X_1 + 0.43\, X_2 + 0.38\, X_3 + 1.13\, X_4 + 0.84\, X_5 + 1.11\, X_6 = 516$$

$$0.70\, X_1 + 0.50\, X_2 + 0.38\, X_3 + 1.25\, X_4 + 1.00\, X_5 + 1.11\, X_6 = 576$$

The initial simplex tableau.

Table 3. *Tableau I (Initial simplex tableau).*

Variable in Solution	X_1	X_2	X_3	X_4	X_5	X_6	S_1	S_2	S_3	Solutionquantity
	18.00	21.60	20.57	19.20	21.60	14.40	1	0	0	3800
	0.55	0.43	0.38	1.13	0.84	1.11	0	1	0	516
	0.70	0.50	0.38	1.25	1.00	1.11	0	0	1	576
Z_j	-2.50	-4.16	-4.16	-8.34	-8.34	-8.3	0	0	0	0

Table 4. *Tableau II (Initial simplex tableau).*

	18.00	21.60	20.57	19.20	21.60	14.40	1	0	0
P_j	0.55	0.43	0.38	1.13	0.84	1.11	0	1	0
	0.70	0.50	0.38	1.25	1.00	1.11	0	0	1
C_j	2.50	4.16	4.16	8.34	8.34	8.34	0	0	0

The initial simplex tableau.

Table 5. *Tableau III (Initial simplex tableau).*

BV	C_B	B			B-1			b
S_1	0	1	0	0	1	0	0	3800
S_2	0	0	1	0	0	1	0	516
S_3	0	0	0	1	0	0	1	567

Test for optimality for all Pj not in solution. I.e. (X_1, X_2, X_3, X_4, X_5, X_6)

ForX_1 = C_B B-1P_j – C_j

(0 0 0) (1 0 0) (18.00)

(0 1 0) (0.50)-2.50 = - 2.50

(0 0 1) (0.70)

For P1 = - 4.16

For P_2 = -4.16

For P_3 = -8.34

For P_4 = -8.34

For P_6 = -8.34

The above table is not optimal

Let X_6 = incoming variable

Then the outgoing variable will be:

{(3800)}{(1 0 0)(14.40) }

Min {b/B-1P_j}= Min{(516)}/{(0 1 0)(1.11) }

{(576)}{(0 0 1)(1.11) }
= Min{264, 465, 519} = 264
It implies that S1 is the outgoing variable
2nd Revised Simplex Tableau.

Table 6. *2nd Revised Simplex Tableau.*

BV	C_B	B			B-1			b
X_6	8.34	14.4	0	0	0.0694	0	0	264
S_2	0	1.11	1	0	-0.077	1	0	223
S_3	0	1.11	0	1	-0.077	0	1	283

Test for optimality for all Pj not in solution i.e. P_1, P_2, P_3, P_4, P_5, P_6
P_1 = (8.34 0 0) (0.069400) (18.00)
(-0.07710) (0.55)- 2.50 = 7.92
(-0.07701) (0.77)
P_2 = 8.35
P_3 = 7.75
P_4 = 2.77
P_5 = 4.16
P_6 = 0.58
At this point the tableau is optimal and hence
Z = C_B B-1b0 = (8.34 0 0) (0.069400) (3800)
(-0.07710) (516)
(-0.07700) (576)
Z = 2200.834.

5.3. Data Analysis

Table 7. *Initial Tableau.*

		X_1	X_2	X_3	X_4	X_5	X_6	S_1	S_2	S_3		$B_{(j)}$
Basis	$C_{(J)}$	2.500	4.160	4.160	8.340	8.340	8.340	0	0	0	$B_{(i)}$	$A_{(j,i)}$
S_1	0	0.108	0.216	0.206	0.192	0.216	0.144	1.000	0	0	38.00	0
S_2	0	0.550	0.430	0.380	1.130	0.830	1.110	0	1.000	0	516.0	0
S_3	0	0.700	0.500	0.380	1.250	1.000	1.110	0	0	1.000	576.0	0
$C_{(j)}$-$Z_{(j)}$		2.500	4.160	4.160	8.340	8.340	8.340	0	0	0	0	

Table 8. *Iteration 1.*

		X_1	X_2	X_3	X_4	X_5	X_6	S_1	S_2	S_3		$B_{(j)}$
Basis	$C_{(J)}$	2.500	4.160	4.160	8.340	8.340	8.340	0	0	0	$B_{(i)}$	$A_{(j,i)}$
S_1	0	0.108	0.216	0.206	0.192	0.216	0.144	1.000	0	0	38.00	197.9
S_2	0	0.550	0.430	0.380	1.130	0.830	1.110	0	1.000	0	516.0	456.6
S_3	0	0.700	0.500	0.380	1.250	1.000	1.110	0	0	1.000	576.0	460.8
$C_{(j)}$-$Z_{(j)}$		2.500	4.160	4.160	8.340	8.340	8.340	0	0	0	0	

Current objective function value (Max.) = 0
< Highlighted variable is the entering or leaving variable >
Entering: X_4Leaving: S_1.

Table 9. *Iteration 2.*

		X_1	X_2	X_3	X_4	X_5	X_6	S_1	S_2	S_3		$B_{(j)}$
Basis	$C_{(J)}$	2.500	4.160	4.160	8.340	8.340	8.340	0	0	0	$B_{(i)}$	$A_{(j,i)}$
X_4	8.340	0.937	1.125	1.071	1.000	1.125	0.750	5.208	0	0	197.9	236.9
S_2	0	-0.509	-0.841	-0.831	-0.000	-0.441	0.263	-5.89	1.000	0	292.4	1114
S_3	0	-0.472	-0.906	-0.959	-0.000	-0.406	0.173	-6.51	0	1.000	328.6	1905
$C_{(j)}$-$Z_{(j)}$		-5.32	-5.22	-4.78	0	-1.04	2.085	-43.4	0	0	1651	
BIG M	0	0	0	0	0	0	0	0	0	0		

Current objective function value (Max.) = 1650.625
< Highlighted variable is the entering or leaving variable >
Entering: X_6Leaving: X_4.

Table 10. *Final Tableau (Total Iteration = 2).*

		X_1	X_2	X_3	X_4	X_5	X_6	S_1	S_2	S_3		$B_{(j)}$
Basis	$C_{(J)}$	2.500	4.160	4.160	8.340	8.340	8.340	0	0	0	$B_{(j)}$	$A_{(j,i)}$
X_6	8.340	1.250	1.500	1.428	1.333	1.500	1.000	6.944	0	0	263.9	0
S_2	0	-0.837	-1.24	-1.21	-0.350	-0.835	0	-7.71	1.000	0	223.1	0
S_3	0	-0.688	-1.17	-1.21	-0.230	-0.665	0	-7.71	0	1.000	283.1	0
$C_{(j)}$-$Z_{(j)}$		-7.93	-8.35	-7.75	-2.78	-4.17	0	-57.9	0	0	2201	

(Max.) Optimal OBJ value = 2200.83.

Table 11. *Summarized Results.*

Variable names	Solution	Opportunity cost	Variable names	Solution	Opportunity cost
X_1	0	+7.925002	X_6	+263.8889	0
X_2	0	+8.3500013	S_1	0	+0.57916671
X_3	0	+7.7534590	S_2	+223.08331	0
X_4	0	+2.7800007	S_3	+283.08331	0
X_5	0	+4.1700010			

Maximized OBJ. function = 2200.834 Liters.

The findings shows that the linear model formulated for the problem is

$$\text{Max } Z = 2.50X_1 + 4.16\ X_2 + 4.16\ X_3 + 8.34\ X_4 + 8.34\ X_5 + 8.34\ X_6$$

$$18.00\ X_1 + 21.60\ X_2 + 20.57\ X_3 + 19.20\ X_4 + 21.60\ X_5 + 14.40\ X_6 \leq 3800 \text{ (raw material constraints)}$$

$$0.55\ X_1 + 0.43\ X_2 + 0.38\ X_3 + 1.13\ X_4 + 0.84\ X_5 + 1.11\ X_6 \leq 516 \text{ (Labour time constraints)}$$

$$0.70\ X_1 + 0.50\ X_2 + 0.38\ X_3 + 1.25\ X_4 + 1.00\ X_5 + 1.11X_6 \leq 576 \text{ (Machine time constraints)}$$

$$X_1, X_2, X_3, X_4, X_5, X_6 \geq 0 \text{ (Hidden constraints)}$$

The computation of the revised simplex method for raw material consumed, production output, labour time, machine time and the resultant profit achievable from all this variables was optimal at the second and final iteration tableau with only SQUASH Drinks as the preferred product that gave the company maximum profit.

The maximum profit (i.e. Z = value) from the analysis is #2200.834 (Two thousand two hundred naira, Eighty-three kobo).

The quantity of Squash drinks that gave the profit of #2200.834k is 264 units produced.

6. Conclusion

Based on the research and analysis carried out, it is observed that The computation of the revised simplex method for raw material consumed, production output, labour time, machine time and the resultant profit achievable from all this variables was optimal at the second and final iteration tableau with only SQUASH Drinks as the preferred product that gave the company maximum profit. It is obvious that linear programming using revised simplex method can be useto solve operational problem of WHAO Beverages Nig. Ltd. It also was found that squash drinks yield maximum profit of #2200.834 for the company for every 264 units of produced.

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